Investigation of chaos in the mechanistic turbulence model

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Abstract. In this work, the chaotic behaviour of the mechanistic turbulence model is examined for free vibrations and harmonic excitations. The multi-degree-of-freedom oscillator is investigated with different numbers of nonlinear springs, and the Lyapunov exponents of the system are calculated in order to determine whether it is chaotic.

Introduction

The mechanistic model of turbulence consists of a binary tree of spring connected masses, with dampers between its last two levels, as shown in Figure 1. The model with only linear springs was found to be capable of producing similar energy spectra to the Kolmogorov spectrum found in 3D homogeneous turbulence [1]. Using nonlinear springs at the bottom level provides a mechanism for targeted energy transfer [2], which efficiently dissipates the energy of the system [3]. A version of the model with nonlinear energy sinks at the last level exhibited a strong dependence on the initial conditions of the system [4]. However, this sensitive behaviour was not yet quantified with Lyapunov exponents, thus the chaoticness of the system was not determined.

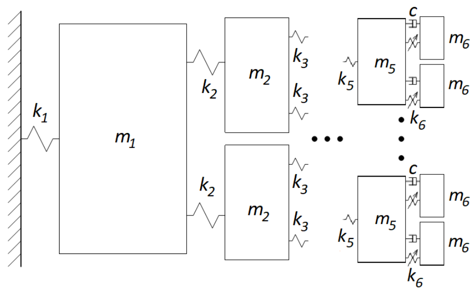

Figure 1: Mechanistic turbulence model with 6 levels

Results and discussion

The system is considered with different number of levels containing nonlinear springs, both with free vibrations and $A\cos(\omega t)$ harmonic forcing of the largest mass. Figure 2 shows the average energy fraction of the system for different ω values. It is expected that chaotic behaviour can be found in the regions where the energy of the last ($j = 6$) level is significant [5]. The Lyapunov exponents are numerically calculated using the method described by Argyris et al. [6], the sign of the largest exponent determines if the motion is chaotic. Preliminary results indicate that with harmonic excitation in the chaotic band, and one or two nonlinear levels, the largest Lyapunov exponent is $\lambda_1 \approx 0$, which means that the system is close to chaotic behaviour.

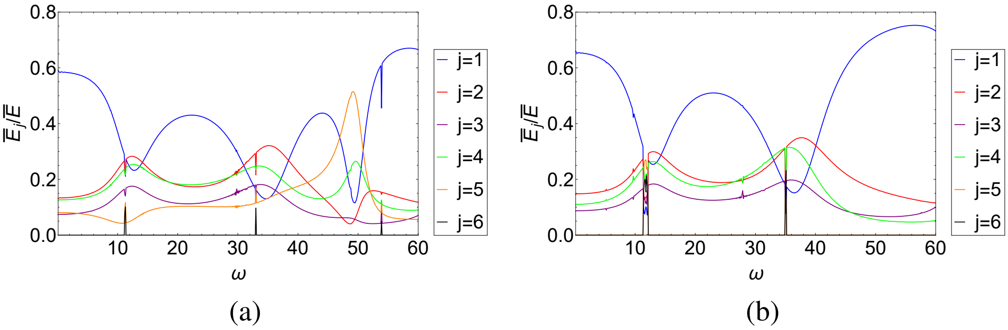

Figure 2: Average energy fraction of the system as a function of ω for (a) 1 nonlinear level, $A = 1$ and for (b) 2 nonlinear levels, $A = 15$

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