# Iterative algorithm for dynamical integrity assessment of systems subject to time delay

**Bence Szaksz***, Giuseppe Habib**

*,** *Department of Applied Mechanics, MTA-BME Lendület "Momentum" Global Dynamics Research Group, Faculty of Mechanical Engineering, Budapest University of Technology and Economics, Budapest, Hungary*

* *ORCID: 0000-0003-1113-0698,* ** *ORCID: 0000-0003-3323-6901*

**Abstract**. The robustness against external perturbations of the desired equilibrium of a dynamical system is an important measure, which can be described by the dynamical integrity. The current study provides an algorithm for determining the dynamical integrity of the equilibrium of systems subjected to time delay. The procedure looks for the local integrity measure, that is, the radius of the largest hypersphere centered in the equilibrium and entirely included within its basin of attraction. The algorithm overlooks possible fractal boundaries and provides a practically reasonable measure with relatively low computational cost.

## Introduction

The stability of an equilibrium state is easy to determine; however, from a practical point of view, its dynamical integrity is also a relevant quantity, the efficient determination of which is still a challenging task. The proposed algorithm estimates the so-called local integrity measure (LIM) of the equilibrium state, that is, the radius of the largest hypersphere centered in the equilibrium and entirely included within its basin of attraction.

## Results and discussion

An algorithm was developed to estimate the LIM of nonlinear ordinary differential equations in [1]. The present work extends it for the case of nonlinear delay differential equations (DDEs) with distinct time delay $\tau$; thus, an infinite dimensional state space should be investigated. The initial conditions of DDEs are functions of time leading to an infinite variety of initial functions with the same headpoint; therefore, the basin of attraction in the space of the physical coordinates can be defined only for specific types of initial conditions. The current work suggests to transform the equations into the space of the modal coordinates and use the free vibration of the undamped linear system as the initial function; however, the proposed algorithm works for other types of initial conditions as well. Another practical question is how to measure the distance in the space of coordinates with different dimensions, for which an energy-based distance definition is proposed.

A semi-discretization [2] based mapping is used to get the trajectories of the solutions corresponding to different initial conditions, which are categorized whether they are converging to the examined equilibrium or not. The diverging category is divided into three subcases: a) when the trajectory exits the predefined space boundary, b) when the trajectory converges to a new unknown solution, or c) when it converges to a periodic solution.

The classification of the trajectories is based on a cell subdivision of the phase space, similarly to the approach utilized in [1]. The difference is that, since the system's state is given by functions and not single points, trajectories are compared with each other as series of cells occupied by their points in the phase space; each series has a length equal to the largest time delay of the system.

The LIM is iteratively reduced if diverging solutions are found until a stopping criterion is met. To get efficient iterations, the simulations' initial conditions are chosen randomly and based on a bisection method. This enables the algorithm to quickly and accurately estimate the local integrity measure. The algorithm was successfully tested on a series of mathematical models, including a delayed Duffing oscillator (Fig. 1), a turning machining model and a delayed controlled inverted pendulum.

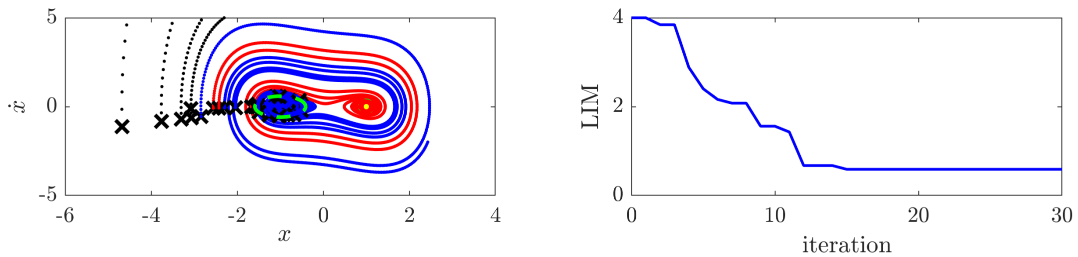


Figure 1: The results of the algorithm applied for the delayed Duffing oscillator $\ddot{x}(t)+0.2\dot{x}(t)+x^3(t) = x(t-0.1)$. Left: trajectories in the state space; converging (blue) diverging (black and red); the green dashed circle indicates the estimated LIM. Right: The estimated LIM during the iteration.

## References

[1] Habib G. (2021) Dynamical integrity assessment of stable equilibria: a new rapid iterative procedure. *Nonlinear Dyn.* **106**(3): 2073-2096.

[2] Insperger T., Stepan G (2011) Semi-discretization for time-delay systems: stability and engineering applications **178** Springer Science & Business Media