

Dynamical behavior of a predator-prey discrete model with prey refuge effect

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Abstract. In ecology, the interaction between distinct species causes conflict, cooperation, and consumption. The prey-predator system is the most fundamental linkage among them. The predator and prey populations had many variations based on their experimental evidence almost a century ago. So therefore, in this paper, a discrete prey-predator model with prey refuge is investigated. A Holling type-II functional response is considered in the interaction term. The discrete model exhibits several bifurcations, viz. flip bifurcation and Neimark-Sacker bifurcation, as the value of one parameter varies using center manifold theory. The conditions of occurrence of the bifurcations are determined. Numerical experiments are done to substantiate the results. The numerical presentation confirms that the discretized model gives more complex dynamics, including limit cycle, period 2, 4 orbits, and chaos, compared to the continuous-time model.

Introduction

In real-world ecosystems and laboratory experiments, the complex phenomena in population dynamics are widely detected [1, 2, 3]. In particular, complex dynamics manifested by the interacting species model have drawn a lot of attention among investigators [4, 6, 8]. Laboratory experiments are useful to provide information about the trait of interaction among species. But mathematically, suitable modeling of the species interaction and analytical study of the models also substantiate the experimental outcomes. This helps us to generalize the results. Therefore the complex dynamical behavior of species and their interactions are widely described by mathematical models, and their analysis [5, 7]. In this paper, section 2 discusses the existence and dynamical behavior of the fixed point of the discrete map. Then different types of bifurcations viz. flip bifurcation and Neimark-Sacker bifurcation (NSB) are exhibited. Extensive numerical experiments are done to validate the analytic findings.

Results and discussion

In this work, a discrete form of a prey-predator model with prey refuge has been investigated. Bifurcation theory and center manifold theory have been employed to exhibit various bifurcations. Neimark-sacker bifurcation and flip bifurcation under certain conditions. The approximate expressions of the above-mentioned bifurcations are also calculated. The existence of different bifurcations implies that both species coexist simultaneously in the system. The numerical simulation extensively presents the occurrence of different bifurcations and chaos.

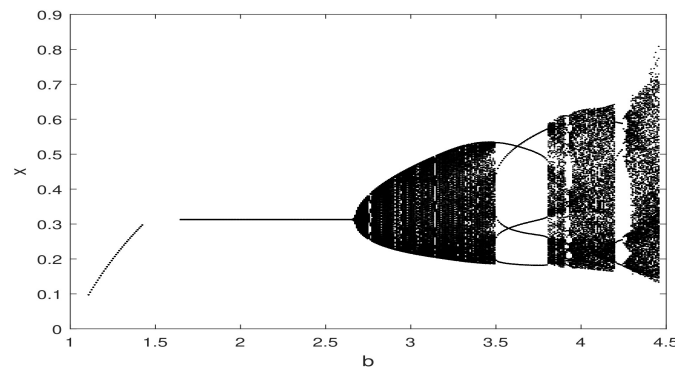


Figure 1: Bifurcation diagram with respect to b for the discrete map

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