Substructural FRF based reduction technique for nonlinear systems

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Abstract. Prediction of the forced response of coupled structures in contact may become computationally expensive due to the nonlinearity in the system. However, the dominant nonlinearity in the coupled structure is localized at the contact interface which gives the opportunity to solve the nonlinear equations only for connection degrees of freedom (DOF) instead of whole structure. In this paper, a new reduction technique for dynamic analysis of nonlinear coupled structure in frequency domain, is presented. The method reduces the number of nonlinear equations to the half of the number of connection DOFs in the coupled structure. Notably, model order substructuring techniques such as fixed, free and mixed interface methods reduce the system to N nonlinear equations, where N is the total number of connection DOFs plus the number of modes included in each substructure. The performance of the method is examined through a case study, and it is shown that the method reduces the computational cost.

Introduction

Most devices contain components that are assembled together for a specific function. Assembling in some applications is inevitable because each component has different material and domain of performance, such as hearing aids which consist of electrical, acoustic and mechanical component. Accurate prediction of dynamic response of coupled structures requires detailed finite element model of each component and nonlinear modeling of the connections which makes it computationally expensive. Despite using high performance computers, developing physics based reduction techniques is necessary to reduce the computational cost.

Component mode synthesis (CMS) methods are one of the well known techniques to reduce the size of the system. These methods can be categorized to fixed interface [1], free interface [2] and mixed interface [3] methods. In all of these methods the system will reduce to the number of connection degrees of freedom plus number of modes included in each substructure. Increasing accuracy of the solution comes with increasing the number of included mode shapes which gives rise to the number of nonlinear equation that should be solved. In this paper we present an approach in which we separate the governing equation of the substructures in frequency domain and using receptance matrix of the each substructure, a close form nonlinear equation is derived as:

$$\mathbf{x}_{rel} = \mathbf{H}^{\mathrm{I}}_{be}\mathbf{f}^{\mathrm{I}}_{e} - \mathbf{H}^{\mathrm{II}}_{be}\mathbf{f}^{\mathrm{II}}_{e} + (\mathbf{H}^{\mathrm{I}}_{bb} + \mathbf{H}^{\mathrm{II}}_{bb})\mathbf{c}(\mathbf{x}_{rel}) \tag{1}$$

where $\mathbf{x}_{rel}$ is the relative displacement vector at the joint and $\mathbf{c}(\mathbf{x}_{rel})$ is the vector of Fourier coefficient of nonlinearity and $\mathbf{H}^{j}$ is receptance matrix of substructure $j = \mathrm{I}, \mathrm{II}$ and indices b and e refer to connection and excitation DOFs, respectively. After solving Eq. 1 using the Newton–Raphson method along with arc-length continuation, the internal force at connection can be calculated. Note that the method does not require the full receptance matrix of the substructure. For calculating the displacements at specific degrees of freedom (m), It is only required to calculate or measure the receptance matrix of the substructures at corresponding degrees of freedom (m), connection and excitation DOFs in the system.

Results and discussion

To investigate the performance of the method, a case study has been considered and the results show that the proposed reduction technique decreases the computational cost comparing to the notable Component Mode Synthesis (CMS) methods, due to excluding the modal amplitudes and halving the connection degrees of freedom in nonlinear equations. Another advantage of the method is that the receptance matrix can be measured or calculated and increasing the number of mode shapes for calculating the receptance matrix, will not affect the number of nonlinear equations in the system. It also gives the opportunity to computed the receptance matrix by expansion based methods such as SOAR [4] and Krylov subspace [5] without any effect on the number of nonlinear equations.

References

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