Phase space visualisation with non-variational chaos indicators

Jérôme Daquin*, Carolina Charalambous*

*Department of Mathematics (naXys), 61 Avenue de Bruxelles, Namur, 5000, Belgium

Abstract. This contribution reports on new global dynamics and non-variational tools able to discriminate between ordered and chaotic motions. The methods are based on geometrical properties of orbits (lengths and stretches), are free of variational equations and are valid for discrete and continuous models. We demonstrate the ability of the proposed indicators to portray Hamiltonian chaos in nearly-integrable settings, and reveal minute details of phase space of various systems in resonant configurations (possibly supporting resonant Arnold web).

Introduction

The problem of discriminating regular and chaotic orbits is of primary importance in several scientific and engineering fields. Over the years, a myriad of computational tools and diagnostics have been developed to achieve this task. These include variational methods relying on the concept of divergence of nearby orbits (*e.g.,* the Fast Lyapunov Indicator [1], the MEGNO index [2] or the SALI [3]), or frequential methods focusing on the spectrum (of some function) of the solution (*e.g.,* the popular frequency analysis method [4]). This contribution reports on easily implementable non-variational and non-frequential methods apt to discriminate chaotic orbits in continuous or discrete dynamical systems.

Results and discussion

Our methods are based on Lagrangian Descriptors [5] (hereafter, LDs), a mathematical and computational technique initially rooted in fluid dynamics, and on the so-called Maximum Eccentricity Method [6] (hereafter, MEM), steaming from the study of exoplanetary orbital systems. Both methods do not rely on variational equations, and exploit solely the knowledge of the orbit. The implementations are thus free of the tangent vector dynamics, and there is no need to quantify its growth over time. Using integrable or perturbed low dimensional dynamical systems (supporting possibly chaotic motions), we discuss properties of the LD and MEM metrics. In particular, by studying how they react on slices of initial conditions, we highlight their dynamical drivers (as exemplified in Fig. 1). The key-point in deriving our chaos indicators then relies on characterising the regularity of the LD and MEM metrics. We demonstrate that second-derivatives based indicators encapsulate sensitively the relevant dynamical information. Our indicators are benchmarked against classical and widely accepted chaos detection methods, and are applied to several models including the standard-map, fundamental models of resonances, symplectic mappings, 3 degrees-of-freedom Hamiltonian models supporting a dense web of resonances, and planetary systems in which diffusion occurs. Our results demonstrate relevance of the indicators for understanding phase space transport mediated by resonant and chaotic interactions, as omnipresent in celestial mechanics or astrodynamics.

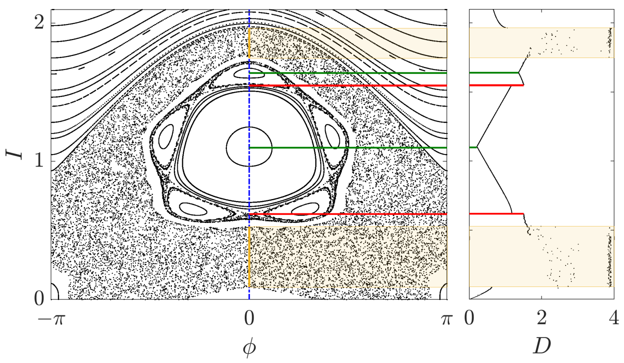

Figure 1: Poincaré map associated to a two-waves Hamiltonian and its associated diameter D metric computed over the line of initial condition $\phi = 0$. The D metric encapsulates relevant dynamical information on which it is possible to build a chaos indicator.

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