Unconditionally stable time stepping scheme for large deformation dynamics of elastic beams and shells

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Abstract. An implicit time-stepping scheme is proposed to achieve energy conservation and unconditional stability for elastic beams and shells undergoing large deformations. More generally, the method can be applied to all structural models, regardless of the nonlinearity in the relationship linking the strain to the kinematic degrees of freedom (displacements and rotations). In this respect, also nonlinear multi-body coupling laws can be included in penalty form by interpreting them as generalized strains. The time stepping scheme is a simple modification of the mid-point rule with the mean internal forces evaluated using the average value of the stress at the step end-points and an integral mean of the strain-displacement tangent operator over the step computed by time integration points.

Introduction

One-step implicit time integration methods such as Newmark's schemes lose the unconditionally stability when used in large deformation problems [1], especially in long simulations. Simo and Tarnow proposed a simple method that guarantees unconditional stability by conserving the algorithmic energy in elastodynamics [1]. However, energy conservation is lost for other structural models where the relationship linking the strain to displacements and rotations is no longer quadratic. This work presents a numerical framework for long term dynamic simulations of elastic structures undergoing large deformations. The time-stepping scheme of Simo and Tarnow is generalized to achieve energy conservation for generally nonlinear strain measures and penalty coupling terms, like the nonlinear rotational one for thin shells [2]. The method is based on a particular integral mean of the internal forces over the step, that includes Simo and Tarnow's method as a reduced quadrature rule, and has unconditional stability.

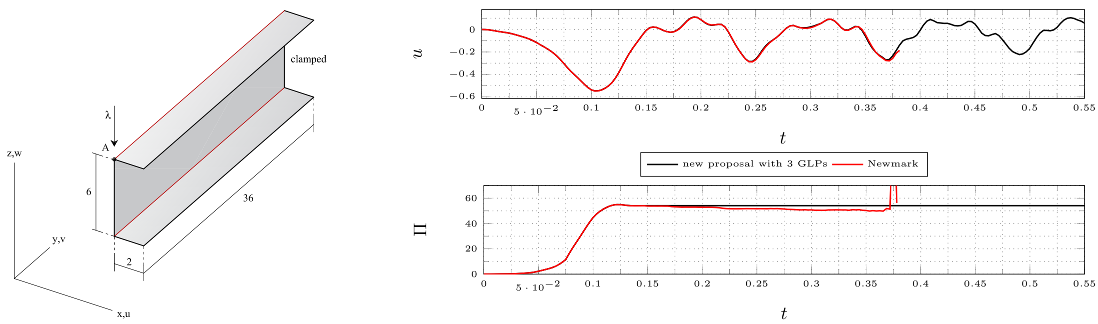

Figure 1: Newmark's trapezoidal rule fails due to the lack of energy conservation while the new proposal is unconditionally stable.

Results and discussion

Numerical results are reported for Reissner beams and assemblages of Kirchhoff-Love shells with smooth and non-smooth interfaces undergoing large deformations. Unconditional stability was proven in long simulations. Compared to the original Simo and Tarnow method, the new one does not conserves exactly the angular momentum. Interestingly, this last feature seems to be marginal in practical computations. Energy conservation assures stability. Conserving the angular momentum neither implies stability nor is synonym of higher accuracy. This is highlighted in the last test, where the momentum-conserving scheme needs a halved time step to get the same accuracy in displacements compared to our energy-conserving scheme. More details are available in [3], together with many other numerical examples. The method is also suitable for models with finite 3D rotations, by exploiting the pseudo-rotation vector.

References

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