Hybrid Autoregressive Neural Networks to predict forced nonlinear Vibrations

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Abstract. In this contribution, we propose a hybrid approach for the data-driven prediction of forced oscillations. The combination of a linear transfer function in frequency domain and an Autoregressive Neural Network (ARNN) is used to model the nonlinear transfer behaviour. For validation purposes, the oscillations of the DUFFING equation, experiencing generic functions as excitation and an automotive use-case based on measurement data, are investigated. Finally, we compare our approach to classic non-hybrid approaches and consider different ARNN architectures.

Introduction

The use of Neural Networks (NN) for the identification and prediction of nonlinear vibration behaviour is an exciting field of research, given the vast number of physical and engineering applications. In particular, for time series prediction, several pure Deep Learning (DL) based architectures have been proposed due to the rapid progress in NN research [1, 2]. Once trained, this NN approach enables low-resource, real-time predictions of system responses. While these architectures perform well for many applications, they often lack generalization capability for unseen input data and longer prediction lengths. Furthermore, they may become problematic when depicting the dynamic oscillating behaviour in forced nonlinear systems [3, 4, 5]. We, therefore, explore a hybrid modelling approach to approximate the nonlinear transfer behaviour between the external excitation and the system response. The main building blocks are the linear transfer function in frequency domain to include the linearized oscillating behaviour of our system into the architecture and an Autoregressive Neural Network (hybrid-ARNN) to account for nonlinear influences. In a two-step process, we firstly approximate the linear transfer function to the given data. Afterwards, the ARNN is trained with the nonlinear system response data using the linear solution and the external excitation as input in an autoregressive setting:

$$\hat{y}(t+1) = \mathbf{y_{lin}}(t+1) + f_{\boldsymbol{\phi}}(\mathbf{x}^s(t+1), \mathbf{y_{lin}}^s(t+1), \mathbf{y_{nl}}^s(t)), \tag{1}$$

where $\hat{y}(t+1)$ describes the predicted system state at timestep $t+1$ while $\mathbf{y_{lin}}(t+1)$ the corresponding linear solution. The external excitation is defined as $\mathbf{x}^s(t)$ where s indicates a sequence (last s time steps) The previous linear and nonlinear system states are defined as $\mathbf{y_{lin}}^s(t+1)$ and $\mathbf{y_{nl}}^s(t)$. Lastly, $f(\cdot)$ defines the mapping by the NN with architecture-specific parameters $\boldsymbol{\phi}$. We compare Feedforward NN, which are easier to handle and can contain basic properties such as symmetry, to gated recurrent NN, which have internal memory to save information over time. All models are trained using adaptive gradient decent techniques in either closed-loop or open-loop environments.

Results and Discussion

We investigate oscillations of the DUFFING equation $\ddot{x} + 2D\dot{x} + x + \alpha x^3 = f(t)$ and an automotive use case based on measurement data. DUFFING system data is synthesised with generic external excitations $f(t)$ (white noise, sweep signals, sine functions) using a DORMAND-PRINCE integration method. The automotive use case extends this one-dimensional theoretical case towards a multidimensional excitation and response for real-world training and validation data from testing tracks. We investigate multiple regularization techniques and show a clear dependency in accuracy due to specific hyperparameters. Our examples emphasize that the proposed hybrid-ARNN achieves significantly higher prediction accuracy than classical linear methods and converges faster than pure ARNN during training. Feedforward NN train faster and achieve superior accuracy in time and frequency domain, while gated recurrent NN suffer from a recency bias in training. Furthermore, we show that a special penalty formulation applied to the weights significantly reduces the training parameters' sensitivity. We conclude that our novel workflow defines a suitable approach for the prediction of forced vibrations in academic examples as well as for a real-world case based on multidimensional measurement data.

References

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