Anomalies in Synchronization of Globally Coupled Mechanical Metronomes

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Abstract. Using a combination of theory, experiment, and simulation, we discover and characterize a new kind of asymmetric long-term dynamic state in a fully symmetric system of four globally coupled mechanical metronomes. In this dynamic state that we refer to as the "Runaway," one of four metronomes settles on an anti-phase relative to the others, and the system randomly "chooses" the identity of the anti-phase metronome. We found that the "Runaway" state can be produced only when the dissipation in the system exceeds a certain threshold. We view this "Runaway" state as a simpler version of a chimera state, in which the symmetry of the oscillator population is broken, but there is no subpopulation with incoherent dynamics.

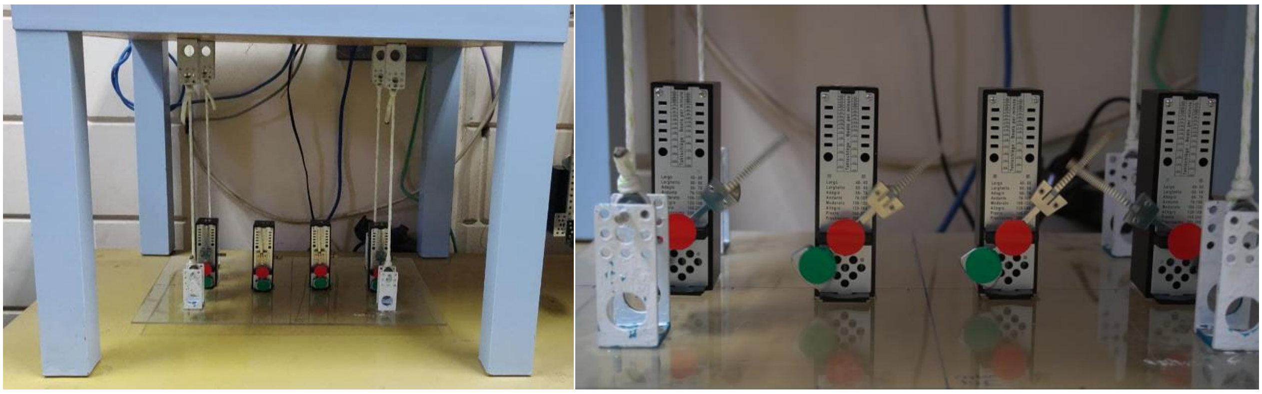

Figure 1: Left panel: The experimental apparatus includes four metronomes docked to a single swing, consisting of an acrylic plate attached to a stationary ceiling via four inextensible ropes and eight pulleys (two pulleys for each rope) to ensure a smooth motion of the swing. The metronomes and the swings are marked with circular colored stickers, and their motion is recorded with a camera. Right panel: Experimental observation of a "Runaway" state. The first metronome from the right is in an anti-phase relative to the other in-phase metronomes.

Introduction

From superconductivity and superfluidity at the subatomic scale to gravitational synchrony at celestial scales, synchronization and collective dynamics pervade all of nature, science, and engineering. In modern technology applications, electronic oscillator networks are frequently used. therefore, their synchronization and collective dynamics are of prime interest, theoretically and practically. Compared to their electronic counterparts, mechanical oscillator networks, such as coupled mechanical metronomes, are considerably simpler to analyse analytically and numerically. Moreover, it has been shown experimentally that mechanical networks of coupled metronomes exhibit a variety of dynamical states, including exotic chimera states [1]. In this study, we explore a simple network of four globally coupled mechanical metronomes (Figure 1, left panel) which obeys the following equations of motion:

$$\ddot{\theta}_i + \sin\theta_i + \dot{\theta}_i\left[2\Gamma_m - J_m\delta(|\theta_i| - \theta_c)H\left(\theta_i\dot{\theta}_i\right)\right] - \frac{2\alpha}{N}\cos\theta_i\frac{d^2}{d\tau^2}\sum_{j=1}^{4}\sin\theta_j = 0, \quad (1)$$

where we used the model in Ref. [2] for the escapement mechanism, $J_m\delta(|\theta_i| - \theta_c)H\left(\theta_i\dot{\theta}_i\right)$.

Results and discussion

We note that Eq. (1) possesses a solution in which $\theta_1 = \theta_2 = \theta_3 = -\theta_4$, i.e., a "Runaway" state. Furthermore, due to the simplicity of the dynamical system, we are able to conduct a stability analysis of the "Runaway" state and find the condition and basin of attraction of this state. These analytical results are validated by numerical simulation and experiments (Figure 1, right panel). Although the synergy between analytical, numerical, and experimental analyses gives us a well-rounded picture of the "Runaway" state, we stress that our study is far from exhaustive, and much still remains to be learned about the long-term dynamics of coupled metronomes, despite the fact that it is considered a simple and well-studied.

References

[1] Martens E. A, Thutupalli S., Fourriere A., and Hallatschek O. (2013) Chimera States in mechanical oscillator networks. *PNAS* **110.26**:10563–10567.

[2] Goldsztein G. H., Nadeau A. N., and Strogatz S. H. (2021) Synchronization of clocks and metronomes: A perturbation analysis based on multiple timescales. *Chaos* **31.2:** 023109.