# Computing Periodic Responses of Geometrically Nonlinear Structures Modelled using Lie Group Formulations

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**Abstract**. This work presents a shooting algorithm to compute the periodic responses of geometrically nonlinear structures modelled using an SE(3) Lie group beam formulation. The formulation is combined with a pseudo-arclength continuation method and used to compute the nonlinear normal modes (NNMs) of a doubly clamped beam. The efficiency of beam model is an advantage that can offset the computational cost of numerical continuation methods. Results are compared with a reference displacement-based FE model with von Kármán strains.

## Introduction

New designs of mechanical structures are increasingly lighter and more flexible and exhibit geometric nonlinearities due to the presence of large displacements and rotations. A popular approach for modelling geometric nonlinearities is to use von Kármán finite element (FE) models, which assume Euler-Bernoulli bending and approximate the Green-Lagrange strain measures by including only the quadratic terms pertaining to the rotations. These methods are widely used for modelling both beams and plates and for creating reduced order models of such structures. However, due to its simplified and approximate treatment of strains and its linearised kinematics, von Kármán equations are not suitable for modelling large deformations. Geometrically exact beam theories can alternatively be used for such cases, however, the parametrisation of rotations can lead to FE discretisations which do not preserve strain invariance under rigid body motion [1]. Other beam models such as the intrinsic beam formulation deals with this issue by eliminating rotations and displacements from the equations of motion, however, they face additional difficulties in FE assembly and in imposing boundary conditions.

Beam formulations based on the Special Euclidean Lie Group *SE(3)* circumvent these problems by coupling the rotations and positions and adopting a local frame approach. The invariance of the strains under rigid body motion comes naturally from this formulation. Moreover, shear locking is avoided thanks to a nonlinear interpolation formula based on the exponential map that couples the rotation and positions fields and governs the nonlinear configuration space [2]. In this work, the Lie group formulation is used to find unforced NNMs of geometrically nonlinear structures, where results are compared to that from a von Kármán beam model.

## Results and Discussion

Results are shown for a straight clamped-clamped beam discretised with 30 elements were NNMs are calculated using the Lie group and von Kármán solvers. The frequency energy plot corresponding to the second NNM is seen in Figure 1. Two resonance tongues appear in the solution curve: the first starting approximately at 160 Hz and corresponding to a 3:1 interaction with mode 4, and the second starting approximately at 207 Hz and corresponding to a 5:1 interaction with mode 6. An additional interaction is captured by the Lie group solver along the first tongue corresponding to an internal resonance between modes 2 and 6, which is not found using the von Kármán solver. The effect of the discretisation on the accuracy of the results is shown in Figure 2, where the Lie group solver can capture the nonlinear dynamics with fewer beam elements, which is an added advantage over the von Kármán solver. Additional structures analysed are a cantilever and an L-shaped beam, where displacements and rotations are larger and the increased accuracy of the Lie group solver in capturing nonlinearities overcomes the limitations of the von Kármán model.

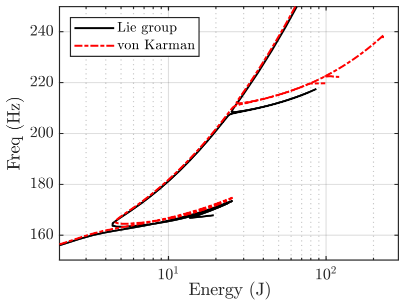


Figure 1: FEP of 2nd NNM of clamped-clamped beam

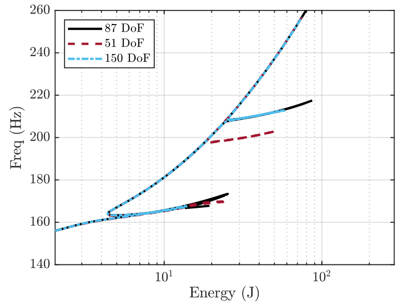


Figure 2: Effect of mesh size on NNM2

## References

[1] Jelenić, G., & Crisfield, M. A. (1999). Geometrically exact 3D beam theory: implementation of a strain-invariant finite element for statics and dynamics. *CMAME* **171(1–2):** 141–171.

[2] Sonneville, V., Cardona, A., & Brüls, O. (2014). Geometrically exact beam finite element formulated on the special Euclidean group SE(3). *CMAME* **268:** 451–474.