Limit cycle bifurcations from infinity in relay systems

Enrique Ponce*, Emilio Freire*, Javier Ros*, and Elisabet Vela*
*Department of Applied Mathematics, E.T.S. Ingeniería, Universidad de Sevilla, SPAIN, ORCID #0000-0003-0467-5032

Abstract. Limit cycle bifurcations from infinity in 3D Relay systems, belonging to the class of three-dimensional symmetric discontinuous piecewise linear differential systems with two zones, are analyzed. A criticality parameter is found, whose sign determines the character of the bifurcation. When such non-degeneracy parameter vanishes, a higher co-dimension bifurcation takes place, giving rise to the emergence of a curve of saddle-node bifurcations of periodic orbits, which allows to determine parameter regions where two limit cycles coexist.

Introduction

The analysis of bifurcations from infinity helps to get a complete overview of the dynamical behaviour to be found in a given dynamical system. Limit cycle bifurcations from infinity has been considered in the past, see [1, 2, 3], but here we want to study the specific case of 3D relay systems, studying also its possible degeneration. Under generic hypothesis, there is no loss of generality in considering relay systems of the form

$$\dot{\mathbf{x}} = A\mathbf{x} - \mathbf{b}\,\mathrm{sign}\left(\mathbf{c}^\top \mathbf{x}\right), \quad A = \begin{pmatrix} t & -1 & 0 \\ m & 0 & -1 \\ d & 0 & 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}, \quad \mathbf{c} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad (1)$$

where $\mathbf{x} = (x, y, z)^\top \in \mathbb{R}^3$, the dot represents derivative with respect to the time τ, and coefficients t, m and d in matrix A are its linear invariants (trace, sum of principal minors and determinant).

Results and discussion

As a first step in the analysis, we present the extension of a previous result in [5] to discontinuous systems (1).
Theorem *Consider system* (1) *under the assumption of complex eigenvalues for matrix A, that is there exist $\lambda, \sigma \in \mathbb{R}$ and $\omega > 0$ such that $t = 2\sigma + \lambda$, $m = 2\sigma\lambda + \sigma^2 + \omega^2$, $d = \lambda\left(\sigma^2 + \omega^2\right)$, and define the non-degeneracy parameter $\delta = b_3 - b_2\lambda - b_1\omega^2$. If $\delta \neq 0$, then, for $\sigma = 0$ the system undergoes a Hopf bifurcation from infinity, that is, one symmetric limit cycle of large amplitude appears for $\delta\sigma < 0$ and σ sufficiently small. Furthermore, when $\lambda \neq 0$, if $\delta > 0$ and $\lambda < 0$, then the bifurcating limit cycle for $\sigma < 0$ is orbitally asymptotically stable. Otherwise, if $\delta < 0$ or $\lambda > 0$ then the bifurcating limit cycle is unstable.*
In the case $\lambda = 0$, assuming $\delta = b_3 - b_1\omega^2 > 0$, a sufficient condition for the stability of the limit cycle that bifurcates for $\sigma < 0$ is $b_1 > 0$.

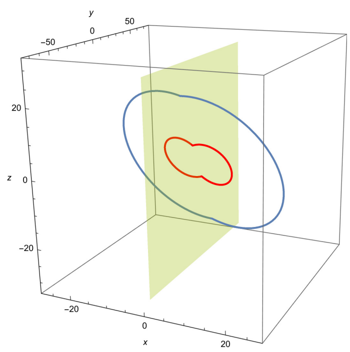

Figure 1: Coexistence of two limit cycles can be guaranteed through a degenerate bifurcation from infinity. Here, $\sigma = -0.01$, $\lambda = -1$, $\omega = 1$, $b_1 = -1$, $b_2 = 4$, and $b_3 = -4$, so that $\delta = 1 > 0$. The big limit cycle is stable, while the small one is unstable.

The case when the non-degeneracy parameter δ vanishes gives rise to a saddle-node bifurcation of periodic orbits and allows to justify the coexistence of two limit cycles, see Figure 1. The application of achieved results to systems in [4] will also be addressed, see [6] for more details.

References

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