Nonlinear stability of a thin viscoelastic film down a vertical wall: A numerical study

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Abstract. We numerically investigate the dynamics of a thin viscoelastic liquid flowing down a vertical wall, based on the earlier study of Cheng et al. (*J. Phys. D: Appl. Phys., vol. 33, 2000, 1674-1682*). They discussed only the linear and weakly nonlinear stability analysis. However, nonlinear effects are more important when the amplitude of the disturbance is small but finite and therefore a nonlinear study is very much essential. We also identify how energy transfers from the basic state to the disturbance in this case.

Introduction

A vast body of thin film literature is devoted to the study of Newtonian films [1]. However, there are various fluids in practice which are non-Newtonian in nature. The viscoelastic fluid is a subclass of non-Newtonian fluids which exhibits features of both ideal fluids (viscosity) and solids (elasticity). Among numerous constitutive models of viscoelastic fluid, the most frequently applied in practical is Walter's B$''$ model as it has only one non-Newtonian parameter through which one can easily obtain a deeper insight of the behaviour of the viscoelasticity on the flow dynamics. The linear stability analysis of a viscoelastic liquid was first discussed by Gupta [2]. He considered a second-order fluid and observed that the viscoelastic parameter plays a destabilizing role on the primary instability. Cheng et al. [3] carried out the linear and weakly nonlinear stability analysis of the viscoelastic liquid flowing on a vertical wall. They found that the viscoelastic parameter has a destabilizing effect on the flow dynamics. In this study, we focus on the free surface evolution equation (37) of [3] to investigate the evolution of the film by numerical simulations. Nonlinear study provides us a first sight of the underlying nonlinear dynamics of the system and helps us to realize the mechanism which is responsible for the transfer of energy from the basic state to the disturbance.

We consider a thin viscoelastic liquid (Walter's B$''$ model) flow on a vertical wall. The interfacial surface of the film is $y = h(x,t)$, where h is the film thickness at any instant t. A highly nonlinear free surface equation $h_t + A(h)h_x + B(h)h_{xx} + C(h)h_{xxxx} + D(h)h_x^2 + E(h)h_x h_{xxx} = 0$ is obtained in [3], where the coefficients $A(h)$ to $E(h)$ are given in [3]. We consider that the initial condition is a simple harmonic disturbance superimposed on the interface as $h = 1 - 0.1\cos(kx)$ in a periodic domain and approximate the spatial solution by a discrete Fourier series. The resulting system of nonlinear ODEs are solved by implicit Gear's method in time with relative error less than 10^{-6}. We also define an energy norm $\mathcal{E}_2 = (1/L)\int_0^L h^2 dx$ to investigate the energy transfer from the base state to the disturbances, where L is the computational domain.

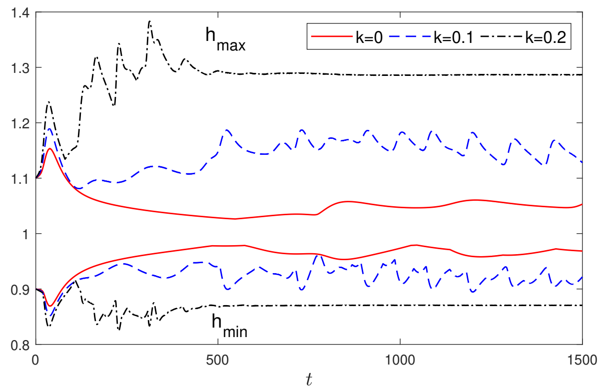

Figure 1: Maximum (h_{max}) and minimum (h_{min}) amplitude of surface wave instabilities for different visoelastic parameter k with fixed $Re = 5$, $S = 12000$ and $\alpha = 0.1$

Results and discussion

1. Viscoelasticity promotes the oscillatory behaviour of the time-dependent wave forms.
2. The surface wave instability for a viscoelastic fluid is larger compared to the Newtonian film.
3. The growth rate of the energy norm is significantly influenced in presence of the viscoelasticity.

References

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