

# An Initial Bifurcation Analysis of an EV Pickup Truck

Shaun Smith\*, Duc Nguyen\*\*, James Knowles\*, Mark Lowenberg\*\* and Sean Biggs\*\*\*

\*Loughborough University, Loughborough, UK, \*\* University of Bristol, Bristol, UK,

\*\*\* Jaguar Land Rover, Whitley, UK

**Abstract.** Vehicles with a beam axle setup, such as trucks, can experience a problem known as “axle tramp”, where the rear axle and wheels undergo a potentially damaging self-sustaining vibration in the vertical, longitudinal and torsional directions. As the automotive industry continues the move towards electrification, it is unknown to what extent adding electric motor(s) to the powertrain changes the problem. The nonlinear nature of the problem make analysis difficult without an efficient method such as bifurcation analysis used in this research. The results characterize an axle tramp region in the electric-drive truck model which is compared to a car with internal combustion engine (ICE). The bifurcation analysis proves to be an efficient way to identify the instability leading to axle tramp, and the harmonically forced bifurcation analysis reveals a further region of tramp not previously observed.

## Introduction

Vehicles with a beam axle setup can undergo a problem known as “axle tramp”, in which the rear axle and wheels undergo a potentially dangerous self-sustaining oscillation, with unwanted motion occurring in the vertical, longitudinal and torsional directions. Previous work on the tramp topic is focused on cars, mainly from the 1960s [1] with few studies relating the work to modern vehicles [2]. With the move towards electrification, it is unknown to what extent tramp will be an issue in larger electric vehicles such as trucks. This study aims develop a low order EV truck model and conduct an initial analysis to determine if tramp exists in the system. The authors propose bifurcation methods as a way to study the problem [3], [4] .

## Results and Discussion

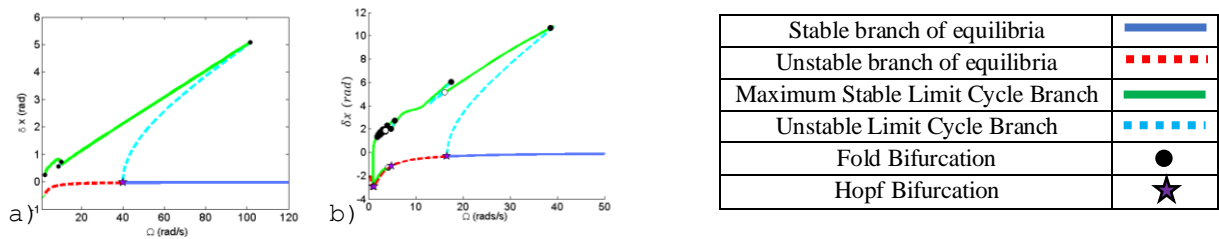


Figure 1. Bifurcation diagrams for: truck (a) and car (b). Angular displacement of the wheel,  $\delta x$ , as a function of engine speed,  $\Omega$ .

Fig 1 presents an initial bifurcation analysis, showing the electric truck model undergoing tramp (a), with an ICE car for comparison (b). The truck undergoes a Hopf bifurcation at  $\Omega=40$  rad/s, with any speed below this threshold causing the system to tramp. A fold bifurcation at  $\Omega=102$  rad/s acts an upper limit on where tramp can be observed depending on initial conditions. The torsional vibration,  $\delta x$ , occurs up to a maximum of 5 radians. The torsional vibrations are smaller than the car but occur over a larger speed range.

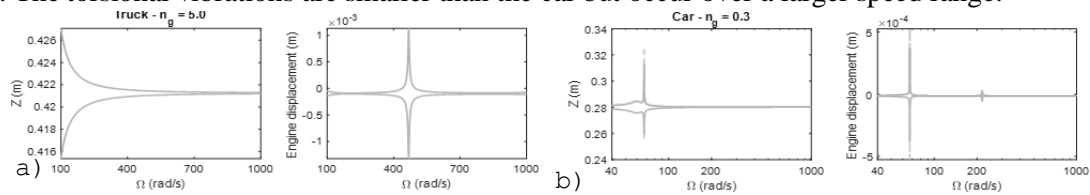


Figure 2. Comparison of the augmented truck (a) and car (b) models. Note that the x-axis starts from the known location of the fold bifurcation in the limit-cycle branch of the unaugmented model.  $Z$  is the vertical displacement of axle and wheel.

The models can be extended to include a simple harmonic forcing term. Fig 2b shows an unstable resonance at 67 rad/s in the car model, in addition to the previously observed result, which leads to tramp at a frequency higher than predicted by equilibrium bifurcation analysis as shown in fig 1b. However, the truck does not exhibit unstable oscillations at high frequencies, even at extreme values of forcing amplitudes (Fig. 2a).

## References

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