Shape Optimization of Curved Mechanical Beams for Internal Resonance Enhancement

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Abstract. In this study, we develop a genetic algorithm-based optimization procedure for coupling enhancement of nonlinearly interacting modes in curved beams. The coupling enhancement leads to more robust internal resonances (IRs) between the interacting modes and can serve as a means to promote directed or targeted energy transfers between the modes. This study aims to obtain the optimal shape function of a curved beam for a specific IR between a pair of vibrational modes. We consider an initial configuration of a curved beam that yields an approximately rational ratio between two eigenfrequencies of the beam and use an iterative genetic algorithm scheme to perturb the shape of the beam and find the optimal curved configuration of the beam that maximizes the coupling between the corresponding modes.

Introduction

IRs are manifested by nonlinear interactions and energy exchange between vibrational modes that arise when modal frequencies of a given structure are (exactly or nearly) rationally related, e.g., $\omega_1/\omega_2 \approx n/m$, where n and m are integers, is the so-called n-to-m IR [1]. The nonlinear coupling between the modes depends on the structure geometry, material, boundary conditions, and transduction schemes. This study focuses only on the structure geometry and how it can be optimized to enhance nonlinear mode coupling and promote IRs.

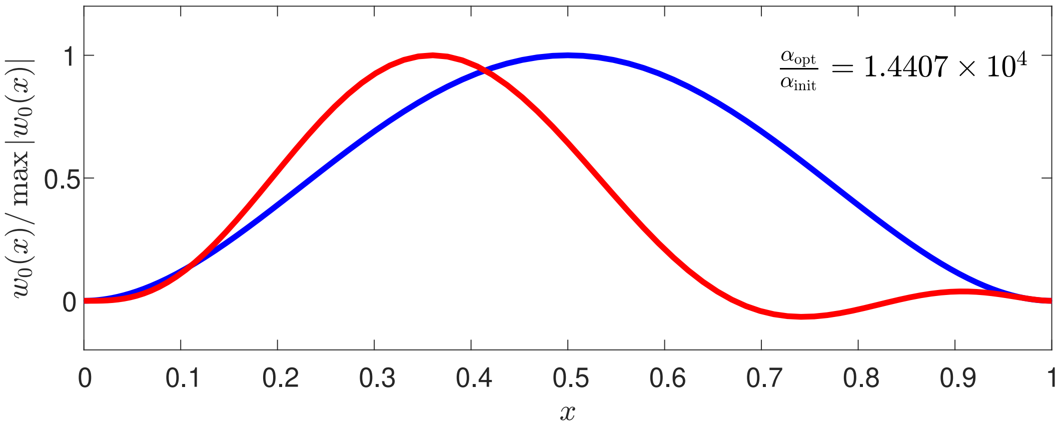

Figure 1: Shape optimization of $w_0(x)$, the initial shape function of the beam, for 1-to-3 IR. Comparison between the normalized initial bell-shaped beam (in blue) and the normalized optimal shape of the beam after 1500 generations of a population of 100 individuals (in red). The 1-to-3 IR stems from a single-term coupling potential of the form $U(x_1, x_2) = \alpha x_1^3 x_2$, where the optimal coupling coefficient α_{opt} is more than three orders of magnitude larger than the initial coupling coefficient α_{init} and the ratio between the eigenfrequencies is $\omega_2/\omega_1 \approx 3.01$.

Results and discussion

We consider IRs that stem from a single-term coupling potential (to leading order) of the form $U(x_1, x_2) = \alpha x_1^m x_2^n$, i.e., the conservative sub-system of the two interacting modes is given by

$$\ddot{x}_1 + \omega_1^2 x_1 + f_1(x_1) = -\partial U/\partial x_1,\ \ddot{x}_2 + \omega_2^2 x_2 + f_2(x_2) = -\partial U/\partial x_2,$$

where $f_{1,2}$ represent single-mode nonlinearities such as Duffing nonlinearity. We apply a genetic algorithm [2] to find the optimal shape of the curved beam that yields an IR condition with the highest (and lowest) coupling term. The genetic algorithm uses the initial shape of the beam to create a population (group of shape functions) of other solutions in its vicinity. After performing a Galerkin projection onto the first two modes of the initial beam, the algorithm calculates the coupling coefficient α for each individual (certain shape function) of the population (given the constraint $|\omega_1/\omega_2 - n/m| < \epsilon$). The algorithm creates the next generation of the population using the fittest solutions of the last generation by: (i) selection, where the fittest solutions survive for the next generation, (ii) crossover, where every two solutions are being used to create a new solution, and (iii) mutation, where some solutions are changed randomly. The algorithm converged to an optimal shape function for the case of 1-to-3 IR after 1500 generations as shown in Fig. 1. Other ratios of $\alpha_{\text{opt}}/\alpha_{\text{int}}$, and initial and final beam configurations for the cases of 2-to-1 and 3-to-1 IRs will be given in the presentation.

References

[1] Nayfeh, Ali Hasan. (2000) Nonlinear interactions: analytical, computational, and experimental methods. New York: Wiley.

[2] Woon, S., Querin, O. Steven, G. Structural application of a shape optimization method based on a genetic algorithm. *Struct Multidisc Optim* **22**:57–64 (2001).