Rectilinear motion of a chain of interacting bodies in a viscous medium

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Abstract. The rectilinear motion of a chain of identical bodies in a viscous medium is considered. Neighboring bodies interact with each other, there are no restrictions on the magnitude of the interaction forces. The problem of moving the system to a given distance under the condition of coincidence of the configuration of the system and of the velocities of the bodies at the beginning and at the end of the movement is solved. A motion is constructed in which the velocity of each of the bodies is piecewise constant, the velocity of the center of mass is constant. Such motion with the maximum velocity of the center of mass is constructed.

Introduction

The rectilinear motion of a system of N identical bodies A_i, $i = 1, \dots, N$, $N \geq 3$ that moves due to the forces of interaction between the bodies is considered, see Fig. 1. Equations of motion of the system are

$$\dot{x}_i = v_i, \qquad m\dot{v}_i = F_i(t) - F_{i-1}(t) + R(v_i), \tag{1}$$

$i = 1, \dots, N$, where m, x_i, and v_i are the mass, the coordinate, and the velocity of body A_i; $F_i(t)$ is the control force acting from body A_{i+1} on body A_i assuming $F_0 = F_N = 0$; $R(v_i)$ is the resistance force of the medium,

$$R(v_i) = -cv_i|v_i|. \tag{2}$$

There are no restrictions imposed on the control forces $F_i(t)$. Thus, it is possible to instantly change the velocities of bodies v_i, redistributing the total momentum of the system. At the initial time instant:

$$x_i(0) = x_i^0, \quad v_i(0) = v_i^0, \quad i = 1, \dots, N. \tag{3}$$

The described system was studied in [1]–[4] and can model a robotic device moving in viscous media. In [4], the stability of the motion of the system was studied. The current study considers the following problem.
Problem. *Move the system of bodies obeying (1)-(3) to a given distance $L > 0$, provided that the configuration of the system and the velocities of each of the bodies are equal at the beginning and at the end of the motion:*

$$x_i(T) - x_i^0 = L, \quad v_i(T) = v_i(0), \quad i = 1, \dots, N. \tag{4}$$

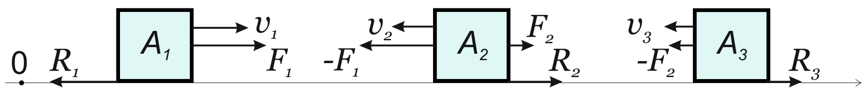

Figure 1: Chain of interacting bodies, $N = 3$.

Results and discussion

The motion that solves Problem is constructed. If at the initial time instant the velocity v of the center of mass of the system is not zero, $v(0) \neq 0$, a motion is constructed in which this velocity is constant, $v(t) \equiv v(0)$, the velocity of each body is piecewise constant and takes values from the set $\{a_i\}$, where these values a_i, $i = 1, \dots, N$ satisfy $\sum_{i=1}^{N} a_i = Nv(0)$ and $\sum_{i=1}^{N} a_i|a_i| = 0$. The velocities of bodies change cyclically:

$$v_i(t) \equiv a_{i+k},\ i = 1, \dots, N-k, \quad v_i(t) \equiv a_{i-N+k},\ i = N-k+1, \dots, N, \quad t \in [t_k, t_{k+1}), k = 0, \dots, N-1.$$

Here, $t_0 = 0$ and $t_N = T$. Additionally, we show how to maximize the velocity of center of mass v under the condition that velocities of all bodies are bounded. It is proved that in optimal motion, part of the bodies moves backward with the maximum allowed velocity, and the other bodies move forward with equal velocities.
If $v(0) = 0$, the motion that solves Problem consists of three stages. At the 1st stage, the bodies are instantly set in motion due to impulse control forces and then they move for a certain time interval with all control forces equal to zero. At the 2nd stage, the motion with a constant velocity of the center of mass is performed. At the last stage, the bodies instantly change velocities, then move freely, and then instantly stop. Thus, in contrary to [1], Problem is also solved for the case, where the initial momentum of the system is zero.
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