Maneuvering a Stick in Three-Dimensional Space Using Impulsive Forces

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Abstract. The problem of maneuvering a stick in three-dimensional space using purely impulsive inputs is considered. A steady motion of the stick is one in which it is juggled between a sequence of configurations rotationally symmetric about the vertical axis; such a motion can be viewed as a periodic orbit. In particular, this work addresses the problem of transitioning from one steady orbit to another. The Impulse Controlled Poincaré Map approach is used to achieve the desired control objective. Simulation results verify the efficacy of the control design for several maneuvers.

Introduction

The robotic manipulation of objects without grasping, *i.e.* nonprehensile manipulation, represents an important class of problems [6, 7]. Juggling is a nonprehensile manipulation task whose dynamics are hybrid and non-smooth, comprised of impulsive dynamics due to contact with the actuator, and motion under gravity. The problem of juggling a stick, which is described by orientation in addition to position coordinates, is more challenging than that of juggling point masses [8, 1]. The dynamic model and control design for planar symmetric stick juggling has appeared in [3, 4]. The problem of juggling a stick in three-dimensions between a sequence of configurations rotationally symmetric about the vertical axis was considered in [5]. This work is extended here to treat the desired juggling motion as a function of time. The dynamics of the stick can be specified by five generalized coordinates, and three control inputs; the control action is purely impulsive and applied when the stick makes a fixed angle with respect to the vertical. The steady-state time of flight and the angle of precession about the vertical axis between consecutive rotationally symmetric configurations are treated as free variables. When they are chosen to be constant, they define a steady juggling motion. They may be varied slowly to transition from one steady juggling motion to another using the same control inputs - which is the focus of this work. The hybrid dynamics of stick juggling can be represented by a discrete-time Poincaré map. The map is conveniently expressed in a rotating reference frame (the reference frame of the juggler), and a steady juggling motion corresponds to a fixed point of this map. The Impulse Controlled Poincaré Map (ICPM) approach [2] can be used to achieve the control objective of stabilization of the fixed point of the map, and consequently the desired juggling motion. By treating the independently chosen steady-state values as time-varying, and recomputing the desired steady-state values and the control inputs at every intersection of the system trajectory with the Poincaré section, the ICPM approach can be applied to achieve the control objective of transitioning from one steady orbit to another.

Results and discussion

Simulations were carried out with the controller gains obtained using the LQR method. Three different maneuvers of the stick, wherein a transition from one steady juggling motion to another is observed, were considered. First, it was shown that the height of the orbit could be changed gradually as a function of time. The effect of an increase in the time of flight was considered next. Finally, the effect of simultaneous variation the angle of precession about the vertical axis and time of flight was considered. It was shown that the ICPM approach can be successfully applied to transition from one steady juggling motion to another. By choosing the free variables as slowly varying functions of time, a range of other maneuvers can be achieved. Future work will focus on experimental validation of steady-state stick juggling.

References

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