Dynamics of a damped variable mass system : Leaky balloon with string

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Abstract. In this work we explore the dynamics of a variable mass system in the form of a leaky balloon with an attached string. Both the interaction of the string with an underlying surface, as well as the variable mass of the balloon itself, contribute to the non-constant mass of the system, which can be treated as a toy model for several other complicated variable mass systems. Our results show that despite being very simple, this system shows a plethora of interesting behaviour that can have important implications for real-world systems such as in stents and in targeted drug delivery.

Introduction

The dynamics of a balloon moving along one direction is non-trivial due to a string attached to it, and by its changing mass due to escape of the gas it contains, two distinct contributions that make for some interesting behaviour. This is a classic example of a variable mass system. We also account for damping effects on the system and investigate its behaviour under forcing. Such a system can act as a model system for various real-world phenomena. In an earlier treatment of a balloon with a string floating in air but without any gas leaking from it, Yariv *et al.* have described the interesting oscillatory motion that it displays, while also drawing an analogy to that of a piston in a fluid [1]. A few other variable mass systems considered in the the literature include those by Denny [2], Bartkowiak and collaborators [3] who investigate a variable mass pendulum, Digilov and Reiner's investigations of a damped, variable mass, spring-pendulum [4], and the several detailed investigations of van Horssen and collaborators of variable mass oscillators (for instance [5]).

Results and Discussion

We present some of the results of our theoretical investigations (both analytical and numerical) of the behaviour of a variable mass balloon-string system near and far from its critical points. We observe how different mass-loss mechanisms and damping methods affect the system. Oscillations in the system die out faster for viscous damping than for quadratic damping. Scaling analysis of the system tells us that although viscous damping can be neglected for certain system parameters, quadratic damping cannot and significantly influences system behaviour. We also discuss how the importance of some system parameters compare to that of the system's dimensions and can be the dominating factor. In addition to Digilov's conclusion that the rate of mass loss significantly affects the system's behaviour, we find that the mechanism of mass loss is also crucial and results in pronounced differences in the leaky balloon system's approach to equilibrium. It is seen that the system retains its robustness even in the presence of external forcing and slowly moves towards a stable attractor, where the rate of attaining the equilibrium state is strongly dependent on the type of damping it experiences. We further try to understand how the ratio of time-scales of balloon oscillation to that of the applied periodic force affects the system. Further details are presented elsewhere [6].

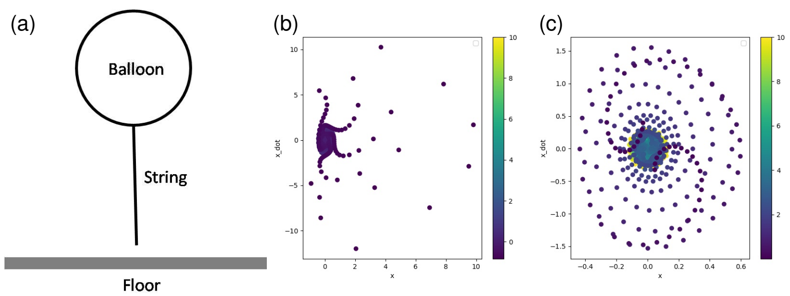

Figure 1: (a) The system, (b) Velocity vs. displacement for leaky balloon, mass-loss indicated by the colour, (c) Details of (b).

References

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