On the stability of sampled-data systems with viscous damping and dry friction

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Abstract. This paper studies the effect of viscous damping and dry friction on the dynamics of sampled-data systems that use discrete-time state feedback in motion control applications. This study aims to present the stabilization effect of Coulomb friction in an otherwise unstable system through the example of a single-degree-of-freedom effective system model. In these systems, unique, so-called concave envelope vibrations occur in cases where the destabilizing effect of sampling can still be compensated to some extent by the presence of dry friction resulting in an unstable limit cycle.

Introduction

One of the fundamental tasks of mechatronics is position control, where the applied controllers aim to drive the system into the desired position. These applications often demand high accuracy and fast operation. However, the necessary performance that can fulfill these requirements may be limited by the presence of physical dissipation and the digital nature of the applied motion controllers.

To illustrate the effect of dissipation, a single-degree-of-freedom mechanical system is considered with viscous damping and dry friction, where discrete-time state feedback with zero-order-hold signal recognition is used to drive the system into the zero reference position. The resulting governing equation of motion is

$$m\ddot{q}(t) + b\dot{q}(t) + f_{\mathrm{C}}\,\mathrm{sgn}\left(\dot{q}(t)\right) = -k_{\mathrm{p}}q(t_j), \quad t \in [t_j, t_j + \tau)\,, \quad t_j = j\tau, \quad j = 0, 1, 2\ldots, \tag{1}$$

where $q(t)$ represents the generalized coordinate as a function of time t, and m denotes the generalized or modal mass. The coefficient of the generalized viscous damping is b and f_{C} denotes the magnitude of the generalized dry friction force. In addition, the parameter k_{p} denotes the feedback gain, t_j represents the jth sampling instant and τ is the sampling time.

Results and discussion

When the system is ideal, i.e., no physical dissipation has been taken into account, the system is always unstable [1]. Suppose that the dominant source of dissipation is modeled by viscous damping. In that case, stable control can be achieved using discrete-time state feedback [2], as presented by the yellow region of the stability map shown in the left panel of Fig. 1. Assume that the dominant source of dissipation can be described by the combined model of viscous damping and dry friction. In that case, the stability region is further extended, but at the same time, it also becomes sensitive to the initial position [2]. It is presented by the shaded gray area of the stability map shown in the left panel of Fig. 1. Numerical simulations and experiments verify these results. To reduce the computational cost and the measurement time, the verification process was carried out by the multidimensional bisection method [3]. The measured stability map is presented in the middle panel of Fig. 1, where the green area represents the stable domain, while the blue area the unstable domain. The comparison of the simulated and the measured stability map is presented in the right panel of Fig. 1.

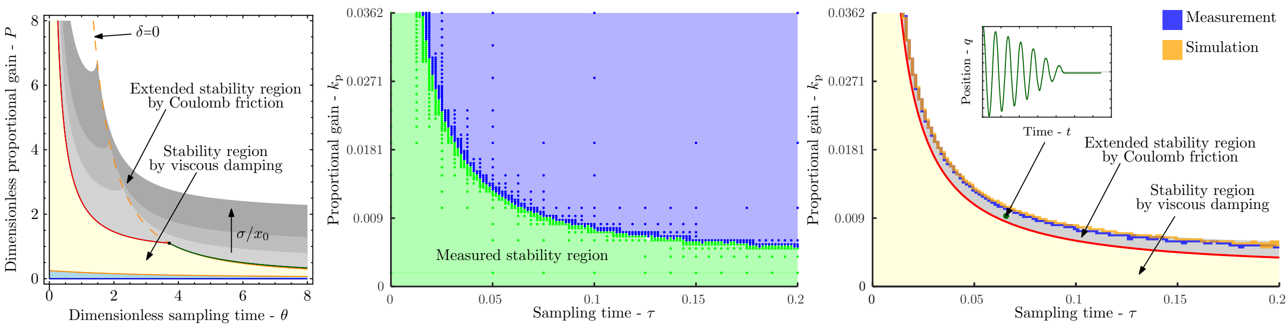

Figure 1: Left panel: Analytical stability chart. Middle panel: Measured stability chart. Right panel: Comparison of the results.

References

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[3] Bachrathy D., Stepan G. (2012) Bisection method in higher dimensions and the efficiency number. *Period. Polytech. Mech. Eng.* **56**(2):81–86.