Softening/hardening dynamics of nonlinear foundation beam with linear stiffening effect

Fangyan Lan[*], Tieding Guo[*], Wanzhi Qiao[**] and Houjun Kang[*]

[*]*College of Civil and Architecture Engineering, Guangxi University, Nanning, China*

[**]*College of Civil Engineering, Hunan University, Changsha, China*

Abstract. Softening/hardening dynamics of a nonlinear foundation beam is asymptotically studied by focusing on its linear stiffening effect. A softening-hardening transition phenomenon is predicted when varying linear stiffness, and in particular, in the vicinity of transition, it is found that standard third-order perturbation analysis fails and a refined high-order (quintic) one is theoretically required to capture essential nonlinear behaviours. Further, a softening-hardening transition inclination concept is defined which turns out to play a key role for the close-to transition nonlinear dynamics.

Introduction

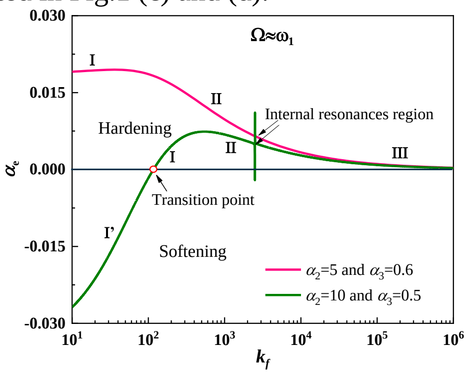
A nonlinear foundation beam is of great interests in both theoretical and application aspects [1], with two competing mechanisms involved, which leads to a typical softening-hardening transition phenomena [2]. We focus on linear stiffening effect on softening/hardening dynamics of nonlinear foundation beam governed by

$$\ddot{w} + w^{(4)} + k_f w + \varepsilon \alpha_2 w^2 + \varepsilon^2 \alpha_3 w^3 + 2\mu \dot{w} = F(x)\cos(\Omega t) \tag{1}$$

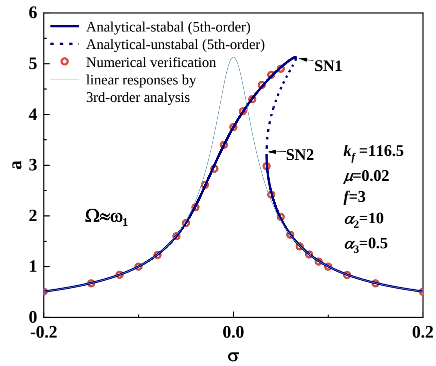
where w is displacement, and k_f, α_2, α_3 are linear, quadratic, and cubic stiffness, respectively, with μ being damping and $F(x)\cos(\Omega t)$ being external excitation. ε is a small parameter.

Results and discussion

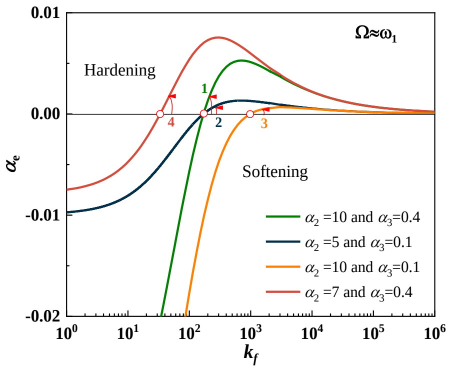
Considering $\Omega=\omega_n+\varepsilon^2\sigma$ and employing a standard 3rd-order perturbation, one can derive a modulation equation with an effective nonlinear coefficient $\alpha_e(k_f, \alpha_2, \alpha_3)$ [1, 2]. α_e dominates the softening/hardening dynamics. Variation of α_e with respect to the linear stiffness k_f is presented in Fig.1 (a). At a critical linear stiffness, $\alpha_e \to 0$, the standard perturbation fails and a high-order one is required as illustrated in Fig.1(b). However, for small transition inclination, standard third-order perturbation analysis at transition turns more reliable as presented in Fig.1 (c) and (d).



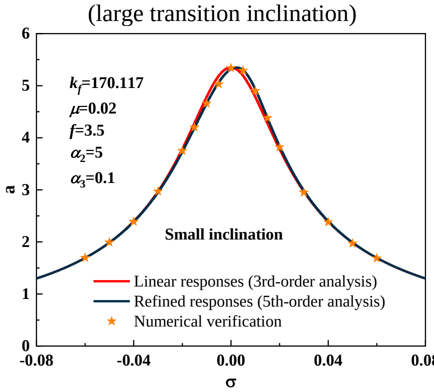
(a) effective nonlinear coefficient α_e vs. linear stiffness k_f



(b) 5^{th}- vs. 3^{rd}-order perturbation at transition (large transition inclination)



(c) α_e vs. k_f with different transition inclinations



(d) 5^{th}- vs. 3^{rd}-order perturbation at transition (small transition inclination)

Figure 1 Linear stiffening effect on softening/hardening dynamics of nonlinear foundation beam

References

[1] Lacarbonara W. (1997) A theoretical and experimental investigation of nonlinear vibrations of buckled beams, Ph.D thesis, Virginia Polytechnic Institute and State University

[2] Lan F., Guo T., Qiao W., and Kang H (2022), An asymptotic study of softening/hardening dynamics of nonlinear foundation beam with a linear stiffening effect, to be submitted