Dynamics of piecewise linear oscillator coupled with wake oscillator

P. Naga Vishnu[*] and K. R. Jayaprakash[*]

[*]*Discipline of Mechanical Engineering, Indian Institute of Technology, Gandhinagar, Gujarat, India*

Abstract. This study is concerned with the analysis of vortex induced oscillations of cylindrical beam with mode-1 crack. The cracked beam is modelled as a piecewise linear oscillator (PWL) coupled with the wake oscillator (Van der Pol (VdP) oscillator). Nonlinear normal modes (NNMs) of this coupled system and their stability are explored using method of averaging and PWL basis functions. The lock-in region (where NNMs cease to exist) is a function of the asymmetry parameter and behaviour of NNMs before and after lock-in are explored.

Introduction

PWL oscillatory models are used for mathematical modelling of several engineering problems like the dynamics of cracked beam [1] [2], mooring towers, suspension bridges etc. Analytical study of PWL oscillator poses challenges owing to their essentially nonlinear behaviour. The study of unforced single degree of freedom PWL oscillator is straight forward as its response can be obtained as a response in two linear regions and matched at the point of transition. However, the study of coupled PWL system poses the challenge of finding the exact transition point. Many researchers have considered both computational and analytical approaches in understanding the complex dynamics of single and two DOF PWL oscillators subjected to external and parametric excitation. Shaw et al. [3] have considered semi-analytical study of harmonically excited PWL oscillator exploring periodic solutions and their bifurcations. Anish et al. [4] studied the resonant response of symmetric, two-DOF PWL oscillator subjected to low-amplitude parametric excitation. In the present study the vortex induced oscillations of a cylindrical beam with mode-1 crack is studied. Due to mode-1 crack, the beam exhibits disparate effective stiffness depending on whether the crack is open or closed. When such a beam is subjected to flow-field, the flow-field couples with the beam and induces vortex induced oscillations which herein is modelled as PWL oscillator coupled with the wake oscillator [5] (VdP oscillator).

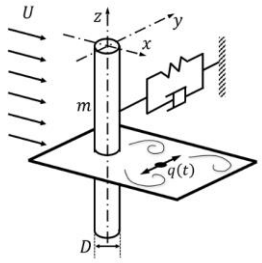

Figure 1: Cylinder in cross-flow (adapted from [5])

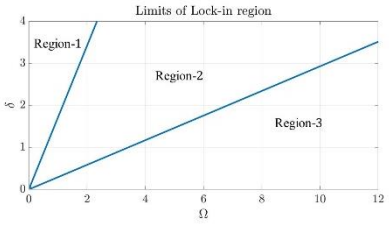

Figure 2: Lock-in region (Region-2) for $A = 12$, $M = 1/24$.

$$\ddot{y} + \varepsilon\lambda\dot{y} + k(y) - M\Omega^2 q = 0$$

$$\ddot{q} + \varepsilon\Omega(q^2 - 1)\dot{q} + \Omega^2 q - A\ddot{y} = 0$$

$$k(y) = \begin{cases} \delta^2 y, y \geq 0 \\ y, y < 0 \end{cases}$$

The fundamental mode of the cracked beam is considered for the analysis and is appropriately inertially coupled with the wake dynamics and the corresponding equation is shown above. Where y is the non-dimensional modal coordinate of the cylinder, q is the ratio of instantaneous and fluctuating lift coefficient, M is the non-dimensional effective mass and is a function of mass ratio, fluctuating lift and Strouhal number, A is the inertial coupling parameter, Ω is the non-dimensional shedding frequency, δ is the asymmetry parameter.

Results and discussion

We study the effect of asymmetry parameter on the dynamics of PWL oscillator coupled with wake oscillator. The lock-in region (Region-2) is observed to be a function of the asymmetry parameter for the unperturbed system. Interestingly, only in-phase NNMs are observed before (Region-1) lock-in and out-of-phase NNMs after (Region-3) lock-in. Since the analytical model is non-analytic due to the PWL function ($k(y)$), direct application of asymptotic methods is seldom possible. We consider method of averaging by invoking PWL basis functions [6] to explore the realization and stability of periodic solutions in the perturbed system.

References

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