Towards the Robust Optimal Design of Nonlinear Metamaterials

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Abstract. In the field of metamaterial design, tailoring nonlinearities can markedly expand the realm of wave manipulations. However, the optimal design of metamaterials with nonlinearities is hampered by a dynamic behavior that exhibits discontinuities and is highly sensitive to uncertainties. For this reason, a specific stochastic design optimization approach is introduced. The algorithm tackles discontinuities and accounts for uncertainties. In addition, the approach introduces a formulation based on a field representation of the design variables, thus reducing the dimensionality of the problem. The stochastic optimization algorithm is demonstrated on the vibration mitigation of a chain of nonlinear resonators. It is shown that the combination of the stochastic optimization algorithm and field representation provides more flexibility in tailoring the chain properties.

Introduction

The design and manufacturing of metamaterials has become an intense research field. From simple band gaps, which prevent the propagation of waves within certain frequency ranges, to the creation of invisibility cloaks, metamaterials offer a host of wave manipulation capabilities [1]. It has long been understood that nonlinearities at the unit cell level can substantially increase the range of possible wave manipulations. Nonlinearities of various forms (e.g., stiffness, constitutive law) have been investigated to achieve specific wave characteristics. Among all the possible promising applications, this study focuses on vibration and shock mitigation.

The design optimization of metamaterials is tedious for several reasons. First, the presence of nonlinearities can lead to discontinuous responses, representative of the high sensitivity of the system dynamics to uncertainties (e.g., material properties, loading). Another difficulty stems from the very large number of units in a metamaterial, which makes optimizing units individually computationally intractable.

Results and discussion

This work introduces the first step towards a general framework for the robust design of nonlinear metamaterials. Specifically, this research focuses on a stochastic optimization approach which accounts for discontinuities detected through clustering [2]. Clusters correspond to various dynamic behaviors and can be mapped onto different regions of the search space, whose boundaries are identified using a support vector machine classifier. The performance of the metamaterial in each region is approximated using Kriging surrogates, which are efficient to propagate uncertainties. The approach was originally developed for nonlinear energy sinks, which exhibit a discontinuous response in the form of an activation threshold [2]. This work also addresses the difficulty due to the dimensionality of the optimization problem (i.e., large number of unit cells) through the use of a field representation. To reduce the problem dimensionality, properties such as the nonlinear stiffness are represented using a spatial field over the metamaterial domain. The field is governed by a handful of stochastic coefficients, thus substantially reducing the dimensionality of the problem. Among the possible problem formulations, the proposed approach is used to optimize the mean of performance metrics such as the RMS response in the case of vibration mitigation. In this study, the optimization of a one-dimensional chain of resonators is considered for demonstrative purposes (Figure 1). The chain, which follows a "mass-in-mass" configuration, is made of units constituted of a main linear resonator and an internal nonlinear resonator. The nonlinearities considered in this study stem from the stiffness properties of the internal resonators. Uncertainties related to stiffness properties, excitation frequencies and amplitude are included in the optimization problem. It is shown that the stochastic optimization is able to tailor the properties of the chain to, for instance, mitigate vibrations. The results also demonstrate the capability of the field approach to reduce the dimensionality of the stochastic optimization problem while achieving performances not achievable with properties constant over the chain.

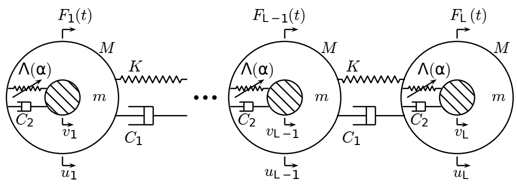

Figure 1: "Mass-in-mass" chain of resonators.

References

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[2] Boroson, E., Missoum, S., Mattei P.O., Vergez C. (2017). Optimization Under Uncertainty of Parallel Nonlinear Energy Sinks. *J. Sound Vib.* **394**: 451-464.