Classification of the post-buckling static and dynamic solutions of a beam under large, but forceless, bending and torsion

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Abstract. The present work proposes a full classification of solutions for beam subjected to bending and torsion at both ends. The formulation is geometrically exact for large three-dimensional transformations, without hypothesis on thickness, slenderness or material. The study is based on analytical and explicit solutions on Kirchhoff rod. This leads on both quantitative and qualitatives observation that could be exploited for a large domain of applications. These observations concern both the static transformation and super-imposed dynamical perturbations.

Introduction

In this study, we are interested in analytical expressions concerning large transformations of a straight beam. No force acts along the beam or as boundary condition. Moment is imposed only at the boundaries. The hypotheses are the following: the beam is elastic, homogenous and has linear constitutive relation. It is shown that under these hypotheses, Timoshenko, Euler-Bernoulli and Kirchhoff beam models lead to the same equilibrium equations. Static solutions are first presented and dynamic behavior super-imposed on these configurations are investigated.

General problem under these hypotheses is presented in a dimensionless form. Geometry and material of the beam are described thanks to three non-dimensional parameters: a slenderness-ratio and two effective rigidities. Bending and torsional moments are encoded by two non-dimensional parameters: the first one is related to the whole energy stored by the structure, the second one is related to the type of moments (according to a predominant torsion or bending). A particular attention is given to the domain of variation of each dimensionless parameter, in order to take into account non-symetric cross-section, thin cross-section and large type of solicitations.

For the static problem, it is shown that two invariants – that depend on the two parameters related to moments and energy density – govern the problem. These invariants are similar to the invariants encountered for finite rotation of a rigid body with the only difference regarding the variables (space in place of time) and the applications between these two different approaches. Accordingly, four regimes arise that depend on the two effective rigidities. Solutions where found in an analytical way, in terms of Jacobian elliptic functions. A detailed discussion is made regarding the role of the control parameters.

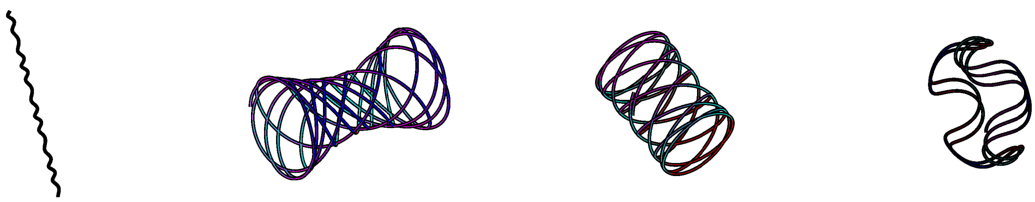

Figure 1: The four regimes of deformation of the beam

Discussion

It is shown that according to the regimes, the beam could roll up in a finite domain whatever its length. The size of such finite domain could be defined in an analytical way. According to relation between the non-dimensional parameters, this process, leads to periodic or only pseudo-periodic patterns.

In a second time, dynamical equations governing the presence of super-imposed infinitesimal perturbations is given thanks the methodology used in [2]. It is the occasion to present the partially coupling of this problem and the various characteristics of the mass and rigidity matrices, involved for such problem. These matrices are space dependent and allow the determination of a stability criteria which is highly dependent on the four regimes encountered in the static configuration. Numerical simulations will follow this presentation.

References

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