Embedding dimension of the dynamical manifold in the phase space as a measure of chimera states

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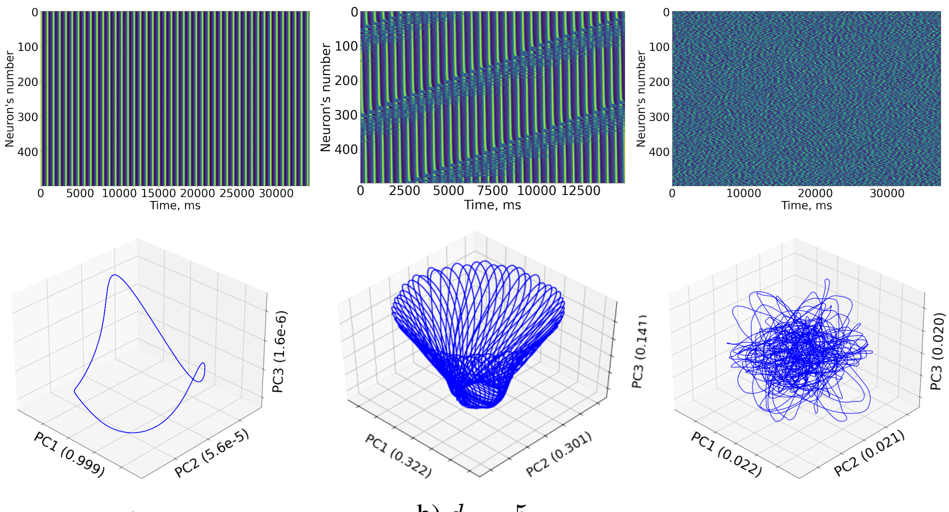
Abstract. The chimera state is a dynamical regime where some elements in a structurally homogeneous network are synchronous while others are asynchronous. Previous studies have shown that even relatively simple networks can have multiple chimera types. Therefore developing methods to identify characteristics of such states has a high potential impact. We propose the embedding dimension as an approach to identify and distinguish chimera states that can be applied to dynamical systems of various natures. The proposed method enables both the accurate identification of chimeras as well as the estimation of the size of their incoherent clusters. To test the new method, we studied a network of type-II Morris-Lecar neurons. with non-local connections. In particular, we considered the evolution of chimera states depending on the parameters of synaptic strength, connectivity, and external current. Using our method, we were able to correctly characterize chimera states and the evolution of their size depending on parameters.

Introduction

Chimera states were first described for a network of phase oscillators [1]. There are now multiple published works that analyze chimeras for a big variety of networks (e.g. see work [2] and refs therein). As a result, several methods have been suggested to determine the characteristics of the chimera states. For example, the Kuramoto order parameter can be used for networks of phase oscillators [1], and the ACM technique can be used for spiking neural networks [3]. In this study, we suggest taking a different angle on this issue based on the idea that network synchronization can be considered as a dimensional reduction of the system dynamics. If all active elements are synchronized, the system activity is effectively equal to the activity of one element. If some of the elements are in an asynchronous regime, then the dimension of the dynamical manifold of the system in phase space has the higher embedding dimension. Moreover, the more there are asynchronous elements in the system, the greater the embedding dimension. Since the chimera states demonstrate partial synchronization, chimera can be identified using the embedding dimension of the dynamical manifold in the phase space.

Results and discussion

We tested a new approach using the same system as in [3]. Examples are shown in Fig. 1.



a) $d_e = 1$ b) $d_e = 5$ c) $d_e = 21$

Figure 1: Examples of different dynamical regimes and corresponding embedding dimension d_e. Top panel: rasterplots of the synchronous state (a),a chimera state (b) and an asynchronous regime (c). Bottom: first 3 PCA-components of the corresponding regimes. Explained variance ratio for each component are given tn brackets.

The advantage of our approach is that it allows for not only the identification of a chimera but also the determination of the "degree of chimera", that is, how large a synchronization cluster is observed in the system for given parameters and initial conditions. This property allows more flexible control of the dynamic regimes in the system, as well as a deeper understanding of how different modulators affect the network.

References

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[3] Dogonasheva O.A., Kasatkin D., Gutkin B., Zakharov D.G. (2021) Robust universal approach to identify travelling chimeras and synchronized clusters in spiking networks *Chaos, Solitons & Fractals* **153**:111541.