A spring-mass mechanical system with moving edges having rich dynamical behaviour

Jobin Josey*[1] and Balakrishnan Ashok*[2]

**Centre for Complex Systems & Soft Matter Physics, International Institute of Information Technology Bangalore (IIITB), 26/C Hosur Road, Electronics City, Bengaluru 560100, India.*

Abstract. We report some of our theoretical results on the behaviour of a mass suspended between two springs and oscillating in a plane with movable suspension points. Our investigations of the system consider the case of springs which are Hookean, as well as when they show nonlinear behaviour, with spring softening and hardening. The effect of external forcing has also been considered, as also the effect of damping in the system. An intensive study of the system has been conducted yielding diverse dynamical regimes, spanning from regular oscillations to chaotic behaviour. The change in stability of the system has been investigated, and the bifurcation mechanisms involved, studied. This study has important applications in various physical systems, ranging from biological systems to mechanical structures in everyday life.

Introduction

The system under study consists of two springs affixed symmetrically to a mass in between them, with the other end of each spring kept fixed to edges that also have a degree of freedom in the same plane. Such mechanical systems are very useful to understand the physical and structural response of other complicated mechanical structures as well as in nature, such as in living systems, including muscle behaviour. Earlier work on such systems include that by Whineray [1], and Arnold and Case [2], who investigates the case of a mass fixed between two springs (a rubber band), but with fixed points of suspension and which are Hookean in nature, the system reduces to a Duffing oscillator under the approximations made therein [2]. Our system is more complicated, and we consider the case of linear and nonlinear springs separately, as also the case where the points of suspension are free to move along an axis in the plane, which has not been treated before in the literature. The form of the nonlinear spring constants is partially motivated by previous work that showed the interaction between carbon nanotubes quantified by the interaction energy obtained through quantum mechanical calculations could be mimicked classically by nonlinear springs [3]. Both forced and free vibrations of the constrained springs are considered, with and without damping, and the equations of motions are solved numerically. The stability of the system is studied intensively, and oscillatory and chaotic regimes are identified. Bifurcation diagrams, Poincare sections, and the largest Lyapunov exponents are calculated.

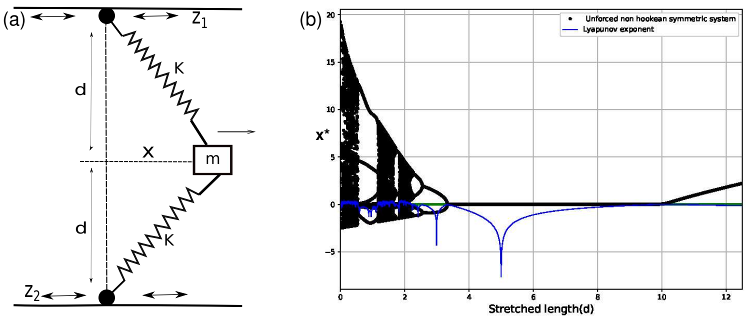

Figure 1: (a) The system, (b) Bifurcation diagram of position $x*$ plotted against stretched length d, also showing the corresponding largest Lyapunov exponent.

Results and discussion

We observe a rich range of dynamics in this deceptively simple system. Regular, oscillatory behaviour is observed, as also quasi-periodic and chaotic regimes. The system has a complex dynamical structure that varies dramatically depending upon the system parameters. The system also behaves distinctly depending upon whether the springs are compressed or elongated with reference to their relaxed lengths. An investigation of the parameter space considering the displacement of the suspension points as parameters reveals a rich portrait of interweaved chaotic and periodic regimes, which change drastically depending upon the value of the relaxed length and the introduction of even very small damping. The stability diagrams of the system with Hookean and non-Hookean springs show some unexpected structures and very distinct appearances. Further details are reported elsewhere [4].

References

[1] Whineray, S. (1991) A cube-law air track oscillator. *Eur. J. Phys* **12**:90-95.

[2] Arnold, Thomas W. and Case, William (1982) Nonlinear effects in a simple mechanical system. *American Journal of Physics* **50**:220-224.

[3] Mishra, B. K. and Ashok, B. (2018) Coaxial Carbon Nanotubes: from Springs to Ratchet Wheels and Nanobearings. *Mater. Res. Express* **5**:075023.

[4] Josey, J. and Ashok, B. (2022) Complex dynamics of a spring-mass mechanical system. (under submission).