# Recursive inverse dynamics of flexible multi-body systems based on Kane's equations

**Pietro Pustina***, Cosimo Della Santina**,*** and Alessandro De Luca *

*Department of Computer, Control and Management Engineering, Sapienza University of Rome, Rome, ITA

**Department of Cognitive Robotics, Delft University of Technology, Delft, NL

***Institute of Robotics and Mechatronics, German Aerospace Center (DLR), Oberpfaffenhofen, GER

**Abstract**. Controlling flexible multi-body systems requires for real-time implementation efficient algorithms that solve the inverse dynamics, an overlooked problem until recently. In this work, we leverage Kane's equations to obtain an inverse dynamics procedure with linear complexity in the number of bodies. The method is simple and coordinate-free because it is independent of the particular coordinates adopted for discretization and inter-body connections. Furthermore, it applies to bodies with lumped and distributed mass and accounts for the changes in the inertial parameters of the system.

## Introduction

Flexible multi-body systems model many processes of practical interest, such as lightweight manipulators, helicopter rotors, and spacecraft. Model-based controllers for these systems are designed upon finite-dimensional descriptions of the dynamics obtained by discretization of the strains. In this way, a finite number of generalized coordinates defines the system configuration. The equations of motion become those of a second-order constrained mechanical system, making the control problem tractable. However, such controllers implicitly rely on efficient inverse dynamics (ID) formulations for real-time implementation. Despite all the effort devoted to solving the forward dynamics, the ID problem has captured far less attention [2]. In the last few years, the soft robotics [4] community manifested a new interest in the topic [3, 5, 6]. Fig. 1 shows a soft robot prototype.

Figure 1: A pneumatic soft robot consisting of three segments. Each segment can bend in any direction by inflating its four air chambers.

## Results and discussion

We exploit Kane's equations to derive an ID procedure for a system of $N$ flexible bodies with a tree topology and a fixed base. These equations, which have been extensively used for modeling flexible systems [1] yield a formulation with an inherent recurrent structure and allow removing nonworking constraint forces. As opposed to [2], the approach does not assume that the body motion can be separated as the sum of a rigid and a deformable contribution. Indeed, this hypothesis is invalid for models obtained from piecewise constant or functional discretizations of the strain fields, which are techniques commonly adopted to formulate control-oriented models of continuum soft robots. In particular, [3] generalizes the RNE algorithm for a geometric model of soft-rigid multi-body systems. Instead, [5] presents a novel formulation of Cosserat beams where the strains are reduced through a functional space representation. Despite being elegant and concise, these approaches require the reader to be familiar with the tools of geometric mechanics and depend on the technique used to discretize the strains. In [6], the authors overcome these limitations by applying the Newton-Euler equations on a lumped mass model of flexible systems. The method is simple but relies on rigid body equations. Consequently, it does not entirely describe the dynamics. Our approach is tractable and independent of the kinematic description of the deformations. Furthermore, each body can have lumped or distributed mass, and it can be connected to its predecessor in the chain by any joint or an hinge. Differently from [6], the resulting equations account for the changes in the inertial parameters of the system. The algorithm also applies to systems with rigid bodies.

## References

[1] Banerjee A. K., Dickens J. M. (1990). Dynamics of an arbitrary flexible body in large rotation and translation. *J. of Guidance, Control, and Dynamics* **13(2)**:221–227.

[2] Boyer F., Khalil W. (1998). An efficient calculation of flexible manipulator inverse dynamics. *Int. J. of Robotics Research* **17(3)**:282–293.

[3] Renda F., Seneviratne L. (2018). A geometric and unified approach for modeling soft-rigid multi-body systems with lumped and distributed degrees of freedom. *2018 IEEE Int. Conf. on Robotics and Automation (ICRA)*, pp. 1567–1574.

[4] Della Santina C., Catalano M. G., Bicchi A. (2020) Soft robots. In: *Encyclopedia of Robotics*. Springer, Berlin, pp. 1–15.

[5] Boyer F., Lebastard V., Candelier F., Renda F. (2020). Dynamics of continuum and soft robots: A strain parameterization based approach. *IEEE Transactions on Robotics* **37(3)**:847–863.

[6] Jensen S. W., Johnson C. C., Lindberg A. M., Killpack M. D. (2022). Tractable and Intuitive Dynamic Model for Soft Robots via the Recursive Newton-Euler Algorithm. *2022 IEEE Int. Conf. on Soft Robotics (RoboSoft)*, pp. 416–422.