Harmonic Scattering of Waves from Crossed-Thin-Rectangular Nonlinear Inclusions

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Abstract. Nonlinear wave manipulations, like the presence and absence of higher harmonics and mode conversions due to harmonic scattering of nonlinear waves from uniquely proposed nonlinear inclusions, are demonstrated in this study using the finite element method due to the limitations of theoretical techniques. The sensitivity of the parameters that control the shape and distribution of nonlinear inclusions is explored to capture possible overall nonlinear effects due to complex harmonic scattering and interference of scattered waves. The exchange of harmonic energies between harmonically scattered longitudinal and transverse waves from multiple nonlinear inclusions is demonstrated. This study will help researchers to design nonlinear metamaterials to control nonlinear waves, as harmonic scattering manipulates nonlinear waves effectively.

Introduction

Interaction of monochromatic (*f*) longitudinal waves with the local single nonlinear inclusion modeled as quadratic and cubic nonlinear material results in harmonic scattering of the nonlinear waves; the theoretical solution is obtained by Kube (2017) using Green's functions [1]. Harmonically scattered waves show the presence of longitudinal and transverse waves and their higher harmonics (*2f*, *3f*) over the 360^0 scattered angles. This understanding is used to quantify highly local early-stage damages using a nonlinear ultrasonic technique [1]. As the number of nonlinear local damages increases, the complexity of multiple interactions of harmonically scattered waves increases, and theoretical techniques fall short of studying such problems. The finite element method is used in this study to understand the complex behavior of harmonically scattered nonlinear waves from multiple nonlinear inclusions of complex shapes. Considering the direction and mode dependency of harmonically scattered waves, a unique shape of nonlinear inclusion is proposed, as shown in Figure 1a, to explore the possibilities of nonlinear wave manipulations. Nonlinear inclusions are modeled as the Murnaghan hyperelastic material. There is no impedance mismatch between the matrix material and the embedded nonlinear inclusions.

Results and Discussion

The interaction of the monochromatic ($f = 2\ MHz$) longitudinal wave with the proposed shape of nonlinear inclusions shows the generation of higher harmonics (*2f* & *3f*), as seen in Figure 2a. The harmonic responses of the received waves at transducer T_2 show that with an increase in the number of embedded nonlinear inclusions, the amplitude of higher harmonics decreases (Figure 1b) when $\theta = 45^0$.

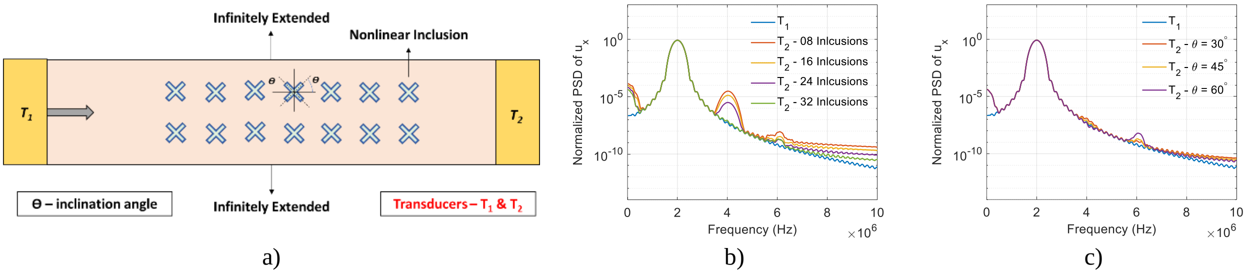

Figure 1: (a) Schematics of wave propagation in linear material embedded with nonlinear inclusions; frequency responses of the waves due to harmonic scattering (b) with an increase in the number of nonlinear inclusions by keeping $\theta = 45^0$ (c) with respect to the change in inclination angle of the crossed thin rectangles by keeping 32 nonlinear inclusions in the domain (T_1 – Input Transducer, T_2 – Receiving Transducer).

As the number of periodically spaced nonlinear inclusions reaches 32 (Figure 1b), 2nd (*2f*) and 3rd (*3f*) harmonics along with the static term (*0f*) go away despite the presence of such a high number of nonlinear inclusions, and this can be referred as an elastically invisible set of nonlinear inclusions. Due to multiple complex phenomena like harmonic scattering, mode conversion, and trapping of harmonically scattered waves in the linear regions because of interference, higher harmonics get canceled as they reach the receiving end. The change in inclination angle (θ) affects the amplitude of the 3rd harmonics greatly, but it's nearly insensitive towards amplitudes of 2nd harmonics (Figure 1c). Results clearly show that we can manipulate nonlinear waves by tuning parameters related to embedded nonlinear inclusions. These computational studies will motivate researchers to design novel nonlinear metamaterials.

References

[1] Kube C. M. (2017) Scattering of Harmonic Waves from a Nonlinear Elastic Inclusion. J. Acoust. Soc. Am., 141(6):4756–4767.