Approximate analytical investigation of the variable inertia rotational mechanism

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Abstract. Rotational inertial mechanisms (RIMs) can create large mass effects that can significantly alter the dynamics of the systems they are attached to. While the most commonly considered RIMs are linear devices, other nonlinear RIMs have been proposed. This work considers the variable inertia rotational mechanism (VIRM). Approximate analytical analysis using the harmonic balance method is used to investigate the dynamics of a system with a VIRM attached. The results of this work show that the VIRM introduces an effective softening nonlinearity dependent on input amplitude and VIRM properties.

Introduction

The inerter is a linear rotational inertial mechanism (RIM) that produces a force proportional to the relative acceleration between the mechanism's two terminals. This mechanism is often realized by using a ball-screw or rack and pinion to convert translational motion to the rotational motion of a flywheel with static geometry. Inerters can be advantageous for structural control applications because they typically have relatively small physical mass but can create significant mass effects in structures. The variable inertia rotational mechanism (VIRM) is a nonlinear RIM in which the provided mass effects vary with system response. The VIRM can be realized by modifying the flywheel of a linear RIM such that it includes slider masses that are positioned in connection with springs and guides that allow these sliders to move radially in the flywheel (see Figure 1-Left). With this type of configuration, the mass effects provided by the VIRM to a system increase with increases in the rotational velocity of the flywheel. While numerical analyses have been performed on systems with the VIRM, its fundamental dynamics have not been explored analytically [1]. This work uses the harmonic balance method (HBM) to perform an approximate analytical investigation of the dynamics of this system.

Results and Discussion

The system considered for this analysis is a single-degree-of-freedom (DOF) primary structure with a VIRM attached (see Figure 1-Center). The EOM for this two-DOF combined system with u as the primary system displacement and x as the slider radial displacement are shown in Eq. (1) [2]. In Eq. (1), m_s, c_s, k_s, and $F(t)$ are the primary system's mass, damping, stiffness, and load, respectively; the VIRM properties of I_0, α, n, m_{sd}, and c_{sd}, are the static rotational inertia, factor relating the linear displacement to number of flywheel rotations, number of sliders, slider mass, and slider damping, respectively, $\dot{\theta}$ is the VIRM rotational velocity, and $F_{bsd}(x)$ is the slider spring force. For this analysis, $F_{bsd}(x)$ includes a main linear stiffness zone and higher stiffness penalty zone to prevent excessive slider displacements. With the HBM, a harmonic force with a given amplitude was used as the load and the response of the system was assumed to be harmonic with unknown amplitude and the same frequency as the load. By plugging in this assumed response into the system EOM and neglecting the higher-order terms, the approximate amplitude of the assumed system response can be solved. An example of the results of the HBM is seen in Figure 1-Right, which shows the response amplitudes resulting from this analysis with different loading frequencies. These results show a softening nonlinearity and bifurcation; however, the stability of each solution would need to be assessed. While softening is still seen, the VIRM properties and load amplitude impact the bending behaviour of the resulting HBM solution.

$$\begin{aligned} &m_s\ddot{u} + I_0\alpha^2\ddot{u} + 2nm_{sd}x\dot{x}\alpha^2\dot{u} + nm_{sd}x^2\alpha^2\ddot{u} + c_s\dot{u} + k_s u = F(t) \\ &m_{sd}\ddot{x} - m_{sd}x\alpha^2\dot{u}^2 + F_{bsd}(x) + c_{sd}\dot{x} = 0 \end{aligned} \tag{1}$$

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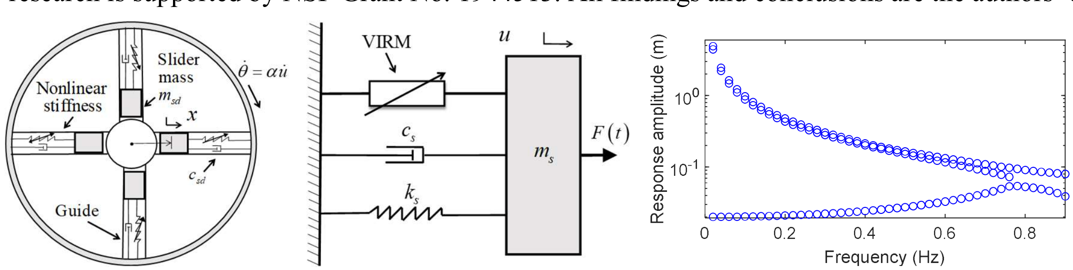

Figure 1: (Left) VIRM flywheel, (Center) Primary system with VIRM, (Right) Example HBM results showing softening behavior

References

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