Saddle-node bifurcation prediction from pre-bifurcation scenario

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Abstract. A technique for the prediction of saddle-node bifurcation is developed. The technique is based on the energy decrement in the free vibration decay, which tends to zero in the vicinity of the bifurcation. This creates a deformation of the energy decrement profile recognizable also very far from the bifurcation, which enables one to estimate the existence and the position of a saddle-node bifurcation directly from the pre-bifurcation scenario in a safer environment from a dynamical integrity perspective. The developed technique is also implementable in experimental systems.

Introduction

Saddle-node bifurcations have the characteristic that the branch of periodic solutions leading to them exists only on one side of the bifurcation parameter. Accordingly, they might be unexpectedly encountered for variations of the bifurcation parameter [1]. A typical bifurcation scenario presenting this situation is illustrated in Fig. 1a. For $a < a_{\mathrm{SN}}$, the only steady-state solution is an equilibrium point, which is globally stable. Conversely, for $a > a_{\mathrm{SN}}$, the system has two additional steady-state periodic solutions, which undermine the dynamical integrity of the stable equilibrium. In this study, a technique for predicting the occurrence of saddle-node bifurcations, which does not require tracking branches of periodic solutions, is developed.

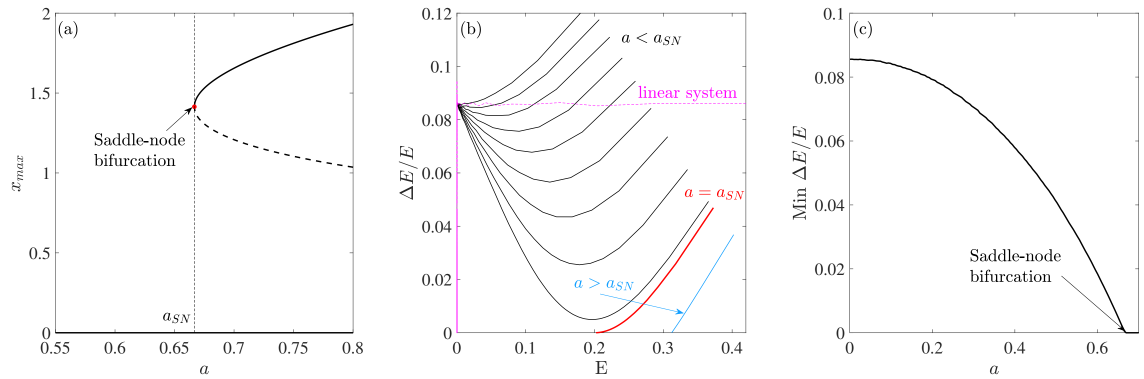

Figure 1: (a) Bifurcation scenario; (b) relative energy decrement per cycle; (c) locus of relative energy decrement minima leading. Results are obtained for the system $\ddot{x} + x + c_1\dot{x} - a\dot{x}^3 + \dot{x}^5 = 0$ [2].

Results and discussion

Let us consider the bifurcation diagram in Fig. 1. For $a < a_{\mathrm{SN}}$, the equilibrium is globally stable. For any initial energy level, the energy monotonously decreases until zero. If the system were linear, the relative energy decrement per cycle would be constant. For $a > a_{\mathrm{SN}}$, the energy decreases until zero within the basin of attraction of the stable equilibrium, while it can either decrease or increase within the basin of attraction of the stable periodic solution, until it converges to the periodic solution at an energy level different from zero. The saddle-node separates these two scenarios. To have a smooth transition between the two scenarios, it is necessary that, starting from $a < a_{\mathrm{SN}}$, the relative energy decrement decreases as we approach the bifurcation and increases moving away from it, which means that the relative energy decrement presents a minimum in the vicinity of the bifurcation, which touches zero at the bifurcation. This conjecture is numerically verified for the considered system, as illustrated in Fig. 1b. The magenta line refers to the linearized system, the black line to the pre-bifurcation scenario and the blue to the post-bifurcation; they are separated by the red one, which touches the zero axes with a horizontal tangent. By picking the value of the minimum points and plotting them with respect to the bifurcation parameter a, we notice that a seeming parabola is obtained. This suggests that, by picking some minimum and performing a second-order interpolation, it is possible to estimate a_{SN} without having any information about the post-critical scenario. We remark that signs that the saddle-node exists are visible already very far from it. This result allows for detecting dangerous saddle-node bifurcations before they are encountered, even in real systems. In fact, the procedure uniquely requires time series of free vibration decays, obtainable numerically but also experimentally.

References

[1] Strogatz S. H. (2018). Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering. CRC press.

[2] Habib G., Cirillo G.I., Kerschen G. (2018) Isolated resonances and nonlinear damping. *Nonlinear Dyn.* **93**:979-994.