Resonant phase lags of an oscillator with polynomial stiffness

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Abstract. Nonlinear systems can exhibit complex behaviours, such as secondary resonances, which are sometimes overlooked in the industry. This work aims at giving a first analytical insight on the behaviour of those secondary resonances and especially their resonant phase lags, nonnecessarily equal to $\pi/2$, using a first-order averaging technique. These phase lags can be later used for experimental nonlinear modal analysis techniques such as phase-locked loop.

Introduction

First- and higher-order avaraging technique is commonly used to describe analytically the behaviour of weakly nonlinear systems [1, 2]. However, the analysis is generaly made for Duffing and Helmholtz oscillators, respectively. The present work extends the existing studies to an oscillator with arbitrary polynomial stiffness:

$$\ddot{x}(t) + 2\zeta\omega_0\dot{x}(t) + \omega_0^2 x(t) + \sum_{d=2}^{\infty} \alpha_d x^d(t) = \gamma \sin \omega t \tag{1}$$

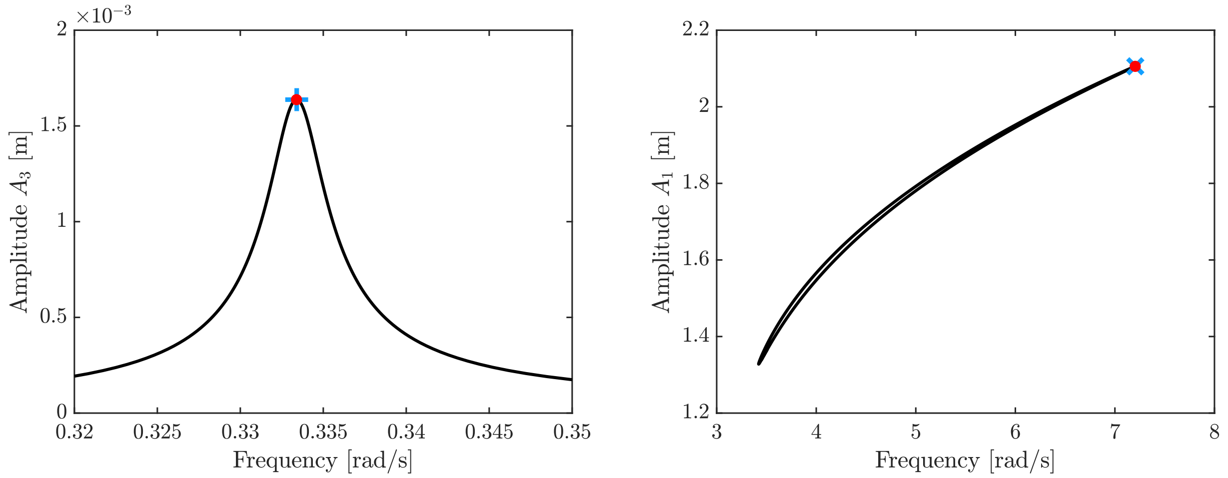
Results and discussion

First, for the primary resonance, assuming small damping ζ, nonlinear coefficients α_d and forcing γ, and writing the solution as $x(t) = A\sin(\omega t - \phi)$. The governing equations show that nonlinearities with even powers do not participate to the motion, and that the phase lag ϕ_a at amplitude resonance is, at first-order: $\tan\phi_a = \frac{\omega_a}{\zeta\omega_0}$, where ω_a is the amplitude resonance frequency. ϕ_a is close to $\frac{\pi}{2}$ since the damping is small. $\phi_p = \frac{\pi}{2}$ is thus defined as the phase lag at phase resonance.

Second, secondary resonances can be studied by assuming small damping and nonlinear coefficients, but strong forcing. The solution can then be expressed as $x(t) = \Gamma\sin\omega t + A_k \sin(k\omega t - \phi_k)$ for $k:1$ superharmonic resonances and $x(t) = \Gamma\sin\omega t + A_\nu \sin(\frac{\omega}{\nu}t - \phi_\nu)$ for $1:\nu$ subharmonic resonances. The governing equations show that nonlinearities with odd (even) powers only generate odd (even) secondary resonances, *i.e.*, when k and ν are odd (even), and for which amplitude resonance occurs close to $\phi_p = \frac{\pi}{2}$ ($\phi_p = 0$), defined as the phase lag at phase resonance for odd (even) secondary resonances.

Higher-order averaging can be used to show that nonlinearities with odd (even) powers do generate even (odd) secondary resonances. For example, even secondary resonances can be found for a Duffing oscillator and the associated phase lag at phase resonance is $\frac{3\pi}{4\nu}$ [3].

These results are illustrated on a $3:1$ superharmonic (Fig. 1a) and a $1:3$ subharmonic (Fig. 1b) where amplitude and phase resonances are shown for an oscillator whose nonlinear term is $\alpha_7 x^7(t)$. Both amplitude and phase resonances occur almost simultaneously for each type of secondary resonance.



(a) (b)

Figure 1: Evolution of the amplitudes A_3 and A_1 around the 3:1 and 1:3 superharmonic and subharmonic resonances (black), phase (red) and amplitudes (blue) resonances

References

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