Synchronised States and Transients in Minimal Networks of Oscillators

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Abstract. Several natural and engineering systems can be modelled using minimal networks of oscillators. The oscillatory behaviour in such networks can be altered by factors such as number of oscillators, coupling scheme, coupling strength and time delay in the system. In this study, we investigate the synchronised states and transients exhibited by a minimal ring of Stuart Landau oscillators. For local coupling, we observe in-phase oscillations and splay state of the highest mode at low coupling strengths. For rings with an even number of oscillators having high coupling strengths, we identify a transient modulated 2-cluster state. We also observe that as the coupling becomes non-local, the variety of splay states increases. For global coupling, we observe generalised splay states. We anticipate that the results of this study will help in the characterisation and control of oscillatory behaviour in physical systems including combustors and coupled lasers.

Introduction

Understanding the dynamics of minimal networks of oscillators is crucial to analyse the behaviour of several systems such as coupled lasers and thermo-fluid systems. A few studies have separately considered the effects of different factors such as coupling scheme, coupling parameters, and number of oscillators [1] on the dynamical behaviour of systems consisting of a large number of oscillators (of the order of 100). Unlike systems with such a large number of oscillators [2], a minimal network of oscillators exhibits noticeable changes in its behaviour as the aforementioned factors are varied. In the present study, we formulate a numerical model of a minimal ring of Stuart-Landau oscillators that helps us to understand the effects of the above mentioned factors on the dynamical behaviour of the ring network.

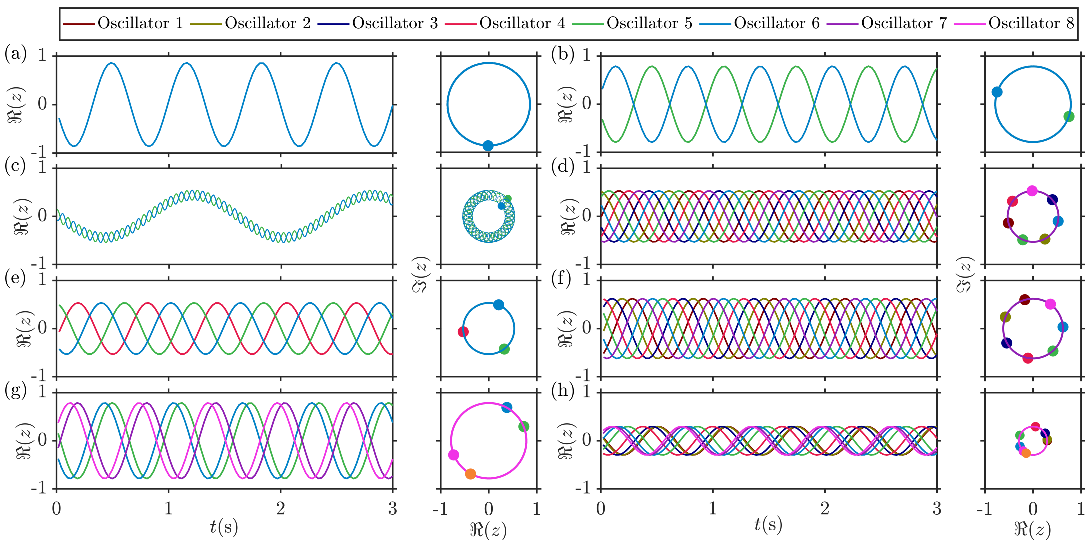

Figure 1: Time series and phase-space representation of various dynamical states (N denotes number of oscillators, K denotes coupling strength, τ denotes time delay. (a) In-phase state ($N = 6$, local coupling, $K = 25$, $\tau = 0.05$), (b) 2-cluster state ($N = 6$, local coupling, $K = 25$, $\tau = 0.30$), (c) Modulated 2-cluster state ($N = 6$, local coupling, $K = 25$, $\tau = 0.15$) (d) Splay state of mode 1 ($N = 7$, non-local coupling, $K = 0.5$, $\tau = 0.15$), (e) 3-cluster state ($N = 6$, non-local coupling, $K = 0.5$, $\tau = 0.25$), (f) Splay state of mode 2 ($N = 7$, non-local coupling, $K = 0.5$, $\tau = 0.25$), (g) Generalised 4-cluster state ($N = 8$, non-local coupling, $K = 0.3$, $\tau = 0.17$), (h) Generalised splay state ($N = 8$, global coupling, $K = 0.5$, $\tau = 0.30$).

Results and discussion

For different combinations of coupling strength, coupling schemes, time delay and number of oscillators, we observe splay states and generalised splay states. In splay states (Fig. 1(a, b, d, e, f, g)), the adjacent pair of oscillators in the ring network exhibit equal phase difference. In a generalised splay state (Fig. 1(h)), the adjacent pair of oscillators exhibit arbitrary phase difference although the order parameter continues to be zero, similar to that observed for a splay state. Additionally, in Fig. 1(c), we observe a transient modulated 2-cluster state characterised by the presence of two dominant frequencies. The lower and higher frequencies correspond to in-phase and 2-cluster states, respectively.

References

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