Parametric resonance and extreme motions of a cantilever with a tip mass: an experimental-theoretical study

Hamed Farokhi[*], Eetu Kohtanen[**] and Alper Erturk [**]

[*]*Department of Mechanical and Construction Engineering, Northumbria University, Newcastle upon Tyne NE1 8ST, UK*

[**]*George W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, 30332, USA*

Abstract. An experimental-theoretical study is conducted on the extreme parametric resonance response of a cantilever with a tip mass. A state-of-the-art in vacuo base excitation experimental set-up is used, consisting of a vacuum chamber and a feedback-controlled long-stroke shaker, together with a high-speed camera to capture large-amplitude motions. A geometrically exact model based on the centreline rotation is utilised for the theoretical part. Extensive comparisons are conducted between experimental observations and theoretical predictions. It is shown that the geometrically exact model's predictions are in excellent agreement with the precisely conducted experimental measurements.

Introduction

Dynamical characteristics of cantilevers have been the topic of investigation by many researchers over the last few decades, with earlier investigations conducted by Crespo da Silva and Glynn [1], who derived the third-order nonlinear equations of motion of a cantilever using the inextensibility assumption, and utilised the method of multiple scales to solve the equations, and Nayfeh and Pai [2], who investigated the lateral vibrations of a base-excited cantilever using a third-order model and the method of multiple scales. Recently, Farokhi et al. [3] proposed a rotation-based geometrically exact model for examining extreme oscillations of cantilevers, which was followed by an experimental study [4] which validated the accuracy of the proposed rotation-based exact model. This study examines the extreme oscillations of an axially excited cantilever with a tip mass in the parametric resonance region for various base acceleration levels.

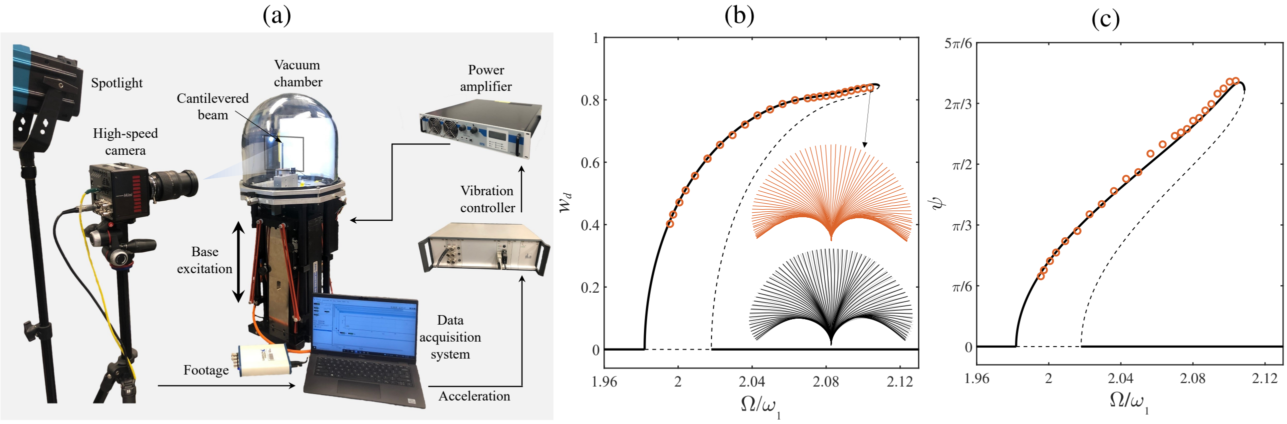

Figure 1: (a) Experimental set-up. (b, c) Parametric resonance responses of the cantilever with the tip mass at $0.25g$ RMS axial base acceleration for transverse and rotational motions at the tip of the cantilever, respectively. Symbols indicate the experimental results, while solid and dashed lines show the stable and unstable theoretical solutions, respectively.

Results and Discussions

Three sets of experiments are conducted at RMS acceleration levels of $0.15g$, $0.20g$, and $0.25g$ using the set-up shown in Fig. 1(a), and detailed comparisons are carried out between the experimental and theoretical results. The results for a representative case are shown here. More specifically, Figs. 1(b) and (c) show the transverse and rotational motion frequency responses for the case of $0.25g$ RMS base acceleration. As seen, the cantilever displays a hardening-type nonlinear behaviour in the parametric resonance region as predicted theoretically and confirmed via experimental observations. Furthermore, it is seen that the geometrically exact model's predictions are in excellent agreement with the experimental results even at this excitation level for which the cantilever undergoes oscillations of extremely large amplitudes. Hence, these comparisons validate the accuracy of the exact model for moderate to extreme vibration analysis of axially excited cantilevers.

References

[1] Crespo da Silva MRM, Glynn CC (1978) Nonlinear flexural-flexural-torsional dynamics of inextensional beams. I. Equations of motion. *J. Struct Mech* **6**:437–448.

[2] Nayfeh, A.H., Pai, P.F. (1989) Non-linear non-planar parametric responses of an inextensional beam. *I. J. Non-Linear Mech* **6**:437–448.

[3] Farokhi, H., Ghayesh, M.H. (2019) Extremely large oscillations of cantilevers subject to motion constraints. *J. Applied Mech* **86**:031001.

[4] Farokhi, H., Xia, Y., Erturk, A. (2022) Experimentally validated geometrically exact model for extreme nonlinear motions of cantilevers. *Nonlin Dyn* **107**:457–475.