# Solving the problem of nonlinear oscillations of a pendulum on a flexible stretchable thread

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**Abstract**. The paper considers the problem of oscillation of a pendulum on a flexible, stretchable thread. The equations of motion are nonlinear partial differential equations. A new method is proposed for solution. The influence of longitudinal and transverse vibrations of the thread is investigated. The transfer of energy from the longitudinal vibrations of the load to the transverse vibrations of the thread is revealed.

## Introduction

Interest in a similar problem arose due to the fact that a nonlinear oscillatory process with several degrees of freedom occurs in vibrations of a pendulum on a stretchable thread . There are works on the vibrations of a pendulum with a spring. The solutions in non-resonance case and asymptotic solutions in resonance were obtained that show there are transfer of energy between vertical and horizontal vibrations. At resonance frequencies ratio regarding horizontal and vertical directions were found to be equal to 1:2. In the paper we discuss the case when longitudinal and transverse vibrations occur in the tread. The equations describing the motion of a pendulum on a flexible elastic thread are nonlinear partial differential equations.

## Results and discussion

The analysis of the incoming dimensionless parameters shows that there are several characteristic times in the system under consideration — the period of oscillation of the pendulum, the time of propagation of longitudinal waves along the thread, as well as the time of propagation of transverse waves. A new asymptotic method is proposed for constructing an analytical solution that takes into account both fast and slow movements when the application of a small parameter leads to a change in the type of partial differential equation. Solutions of the system of equations are obtained. The influence of natural oscillations of a spring pendulum on longitudinal and transverse vibrations of the thread is investigated. A numerical solution of this nonlinear system of equations is also obtained and compared with an analytical solution. It is shown that a pendulum system on a flexible elastic thread is characterized by a motion characteristic of a swinging spring when the following conditions are occurred: a small ratio of the mass of the thread to the mass of the load, no tension drop to zero and no initial nonlinear disturbances. In the first approximation, frequency corrections to the oscillations of a swinging spring are found, which arise if wave processes in the thread are taken into account. It is obtained that the lower the tension, the greater the amplitude of the transverse vibrations of the thread. In particular, the drop in tension leads to instability of oscillations along one axis. The transfer of energy from the longitudinal vibrations of the load to the transverse vibrations of the thread is revealed. In the numerical experiments carried out, it was possible to record a significant increase in the amplitude of the transverse deviations of the thread. It is analytically shown that the parametric resonance between the natural oscillations of the pendulum and the longitudinal-transverse oscillations of the thread differs from the classical one (described by the Mathieu equation), since the frequencies of these oscillations have a different order. The influence of natural oscillations of a spring pendulum on transverse vibrations of a thread at non-resonance and resonance is shown in Fig. 1.

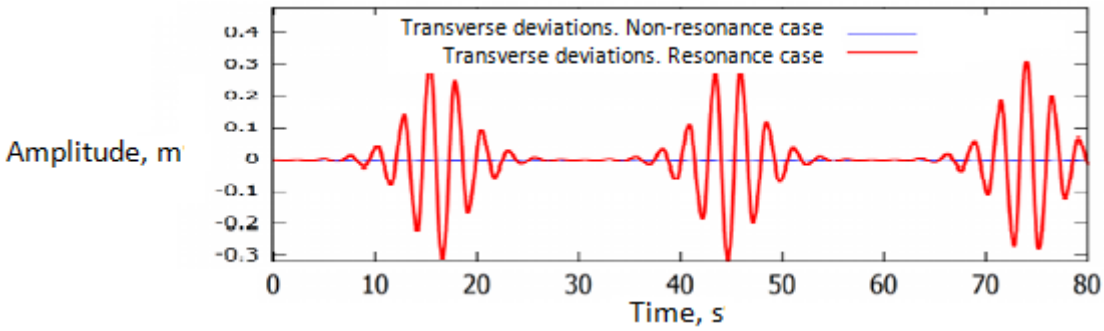


Figure 1: Amplitude of transverse vibrations of the tread of a pendulum with stretchable thread.

## References

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