Time-optimal control approximation for a discontinuous capsule drive

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Abstract. Research connected with capsule drives seem to attract an increasing attention among scientists. In order to assure efficiency of these devices, it is necessary to investigate optimal control for such systems. In this work, authors test a hypothesis that bang-bang control of the capsule drive can be the time-optimal one. For this purpose, a novel numerical method for bang-bang control parametrization and optimization has been developed. Results of its application are compared with a general-purpose algorithm, which approximates optimal control without any assumptions concerning the shape of the control function. It has been shown that the bang-bang control optimization yields 8% increase of the average speed of the capsule.

Introduction

The optimal control problem (OCP) is to find a way of performing a desired task in a given environment at the lowest possible cost [1]. A particular kind of OCP is the minimum-time problem, in which the target is execution of the expected task in the shortest possible time. In a large subset of such problems, the optimal solution is a so-called "bang-bang" control, i.e. a control function that attains only extreme (maximal or minimal) values from the whole set of allowable controls [1]. Unfortunately, although this fact is proven for a class of smooth systems, is not necessarily true if the controlled object is non-smooth or discontinuous. For instance, one cannot be sure whether the bang-bang control is time-optimal for a capsule drive (Fig. 1a). This system is able to move without external moving parts, solely due to interactions between dry friction on the underlying surface and oscillations of the pendulum located inside the device. However, although the dry friction enables motion of the capsule, it is also the cause of discontinuity in the system and the reason for which the assumption of the bang-bang control optimality cannot be a priori assumed. Nevertheless, the fact that discontinuous dry friction can be well approximated by smooth friction models [2] yields a proposition that the bang-bang control may be optimal for discontinuous capsule drive too. Such hypothesis has been tested numerically. In order to perform the comparison, a novel numerical method for bang-bang control parametrization and optimization has been developed. Results of execution of this algorithm have been compared with effects obtained by means of a more general, Fourier series based numerical method [3], which is able to approximate optimal control of an arbitrary shape.

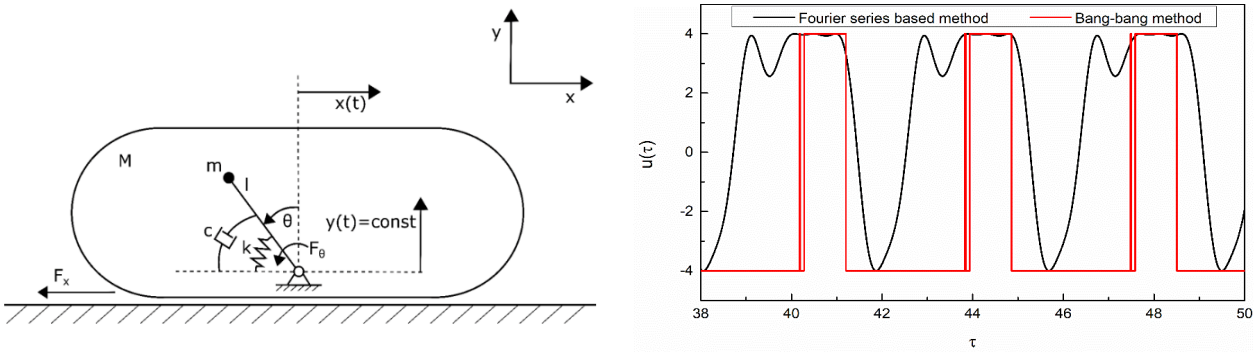

Figure 1: a) Capsule drive scheme (at the left), b) comparison of the optimized control functions (at the right)

Results and discussion

The control function obtained with use of the novel bang-bang optimization method enabled the capsule drive to move with 8% higher mean velocity comparing to the case of optimal control approximation with no restrictions of its shape (Fig. 1b). The obtained results show that restriction of the allowable controls set to bang-bang functions increase efficiency of the system and reduce optimization effort. They can also be regarded as an indication that the bang-bang control may be the optimal one for some discontinuous systems.

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