On the non-trivial solutions and their stability in a two-degree-of-freedom Mathieu-Duffing system

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Abstract. The Mathieu-Duffing equation represents a basic form for a parametrically excited system with cubic non-linearities. In multi-degree-of-freedom systems, other interesting instability conditions take place and considered in this work. Accordingly, the non-trivial solutions were obtained, especially when the trivial solution is unstable. At resonant frequencies a bifurcation analysis was carried out using the multiple scales method, followed by investigating the effect of an asynchronous parametric excitation. Since various micro- and nano-systems include cubic non-linearities and subjected to parametric excitation, this work should be of relevant importance.

Introduction

Over the decades, the solutions of the nonlinear Mathieu equation were studied for stability. In a nonlinear system a parametric resonance could lead to stationary non-trivial solutions and limit cycles, in which the amplitudes are governed by the system's nonlinearities [1]. This happens to be of great interest to systems seeking high amplitudes and thereby possible amplification of input signals. Thus, parametric resonances and amplification were automatically exploited to increase the sensitivity of micro- and nano-sensors [2]. Being more common than others in micro- and nano-systems, a special concern is given to cubic nonlinearities. In this case, the equation is referred to as the Mathieu-Duffing equation [1]. However, a lesser effort was given to corresponding multi-degree-of-freedom (M-DoF) systems, in which the added degrees of freedom introduce other resonant frequencies. In addition, controlling the phase of the excitation terms could lead to a broadband parametric amplification, which was found to be useful in microsystems [3].

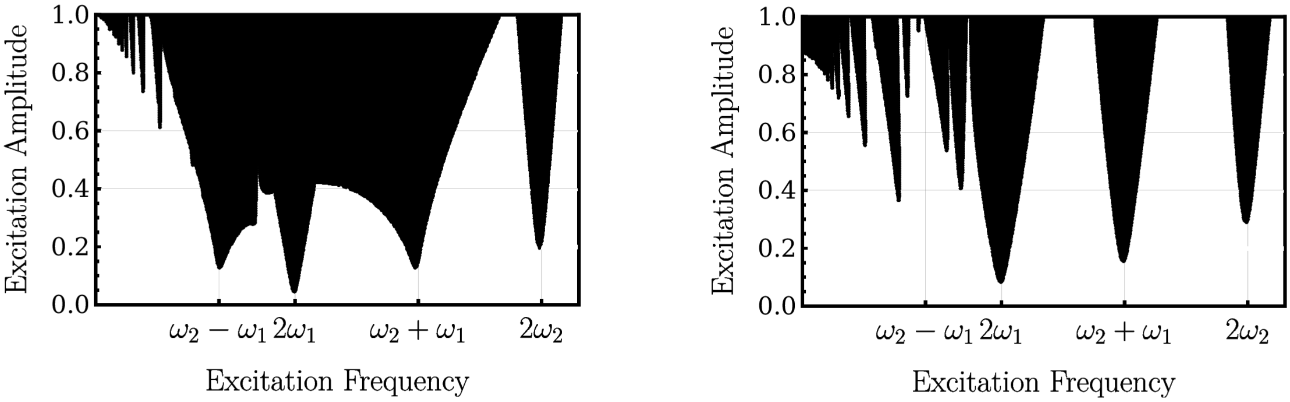

Figure 1: Stability chart: with (left) and without (right) phase-shifted excitation. $\omega_{1,2}$: natural frequencies.

Analysis and results

The trivial solution of the system was studied for stability using Floquet theory and the Lyapunov characteristic exponents were deduced. The resulting stability chart shows the existence of a broadband instability effect under phase-shifted excitation (see Fig. 1, black means unstable). The non-trivial solutions were then obtained in the general excitation case, and especially when the trivial solution turns to be unstable. At resonant frequencies a bifurcation analysis was carried out using the multiple scales method. At the primary resonant frequency the coupling remained ineffective, and a stable limit cycle is detected. However, by controlling the linear damping as a bifurcation parameter, non-trivial stationary solutions were found in a limited domain, exhibiting isolated solutions if the excitation frequency is detuned. By enforcing a one-to-one internal resonance, the energy could be transferred between both degrees of freedom. This transfer of energy occurs also at combination resonant frequencies. Moreover, introducing a phase-shift in the parametric excitation coupling leads to limit cycles at the difference combination frequency. The observed non-linear effects, therefore, show a notable influence on the dynamics, which could be worthwhile for micro and nano-systems using the proposed excitation method.

References

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