Structural reliability analysis based on the dynamic integrity of an attractor

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Abstract. This paper addresses the reliability analysis of a dynamic system attractor, provided its dynamic integrity measure has been previously assessed in terms of a parameter for which the probability density function is known. The probability that the dynamic integrity measure should be equal or larger than a prescribed safe reference value, for the attractor to be considered "reliable", is determined by a simple procedure. Application to an illustrative example is addressed. It is expected that such a simplified reliability analysis may be useful to improve current structural engineering design practices.

Introduction

Although the ideas discussed herewith may be applied to dynamical systems in general, the structural stability case is focused. The proposed concept of dynamic integrity [1,2] applied to buckling analysis has meaningfully improved the definition of a safe load. In fact, it is already well established that the threshold defined by the critical load of the so-called 'perfect' system (let's call it Euler's load) may be unsafe due to its potential imperfection sensitivity, resorting instead to a lower value (let's call it Koiter's load). Nevertheless, even this load may not be an adequate estimate of the safe load, since the associated attractor may have a small or even fractal basin of attraction, so that an even lower value (let's call it Thompson's load) should be considered for adequate engineering design. A dynamic integrity measure (e.g., GIM, LIM or IF) [1,2], to which we will generically refer to as I, seems to be a convenient way to define a safe design load, provided a minimum reference value (I_{ref}) is established. Yet, it is still missing in the state of the art of engineering practice a meaningful reliability measure, such as the probability that the dynamic integrity measure should be equal or larger than that prescribed safe reference value. This is what this paper intends to address.

Methodology

A methodology is proposed considering that the Dover Cliff profile [1] for the dynamic integrity measure I has been characterised as a function of a system control parameter A (for example, the load in a buckling analysis), according to $I(A)$, so that $tan\alpha = -\frac{dI}{dA}$ is the local slope of the Dover Cliff profile. Supposing that the parameter A is a Gaussian random variable, with a standard deviation σ_A about the expected value $\bar{A}$, it is assumed that the output integrity measure will also be a Gaussian random variable with a local standard deviation $\sigma_I = \sigma_A \, tan\alpha$ about the expected value $\bar{I}$. Hence, for every point $(\bar{A}, \bar{I})$ of the Dover Cliff profile, it can be defined the cut-off region for which the integrity measure complies with $I \geq I_{ref}$, provided $A \leq A_{ref}$, leading to the probability assigned for safety. For the sake of an illustration, this methodology is applied to the Dover Cliff profile of the archetypal model discussed in [3], with $I = GIM$ and $A = p$, in which $tan\alpha \cong 2.5$ for the Thompson's load $p_T \cong 0.245$ and $GIM_T \cong 0.100$, as shown in Fig.1. Assuming, for the sake of an example, a standard deviation $\sigma_A = 0.040$, the estimated output standard deviation would be $\sigma_I = 0.100$, leading to a probability of 31.73% for $GIM \geq GIM_T + \sigma_I = 0.200$ if $p \leq p_T + \sigma_A = 0.285$; a probability of 50% for $GIM \geq GIM_T = 0.100$ if $p \leq p_T = 0.245$; and a probability of 68.27% for $GIM \geq GIM_T - \sigma_I = 0.000$ if $p \leq p_T - \sigma_A = 0.205$. These results could be used to decide whether the choices of $GIM_T \cong 0.100$, and henceforth $p_T \cong 0.245$, were good enough for a safe engineering design.

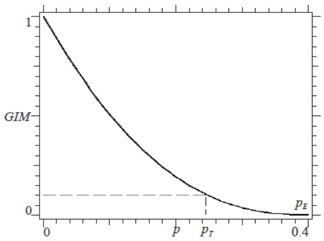

Figure 1: Dover Cliff profile extracted from Fig.8 of [3].

References

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