

# $\phi^4$ equation as a phenomenological model of fronts in microrheology.

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**Abstract.** We propose a numerical model to simulate the fluid-air interface (front) dynamics of a Newtonian fluid in a microchannel in front of a notch. We model the interface using a  $\phi^4$  model with a spatially localized force  $F(x, y)$  to mimic the imperfection. To model a front with curvature, we propose a damping force proportional to the square distance perpendicular to the propagation direction front.

## Introduction

Our motivation is two experimental research: the first was carried out by Trejo-Soto *et al* [1]. They study the behavior of Newtonian fluids of different viscosities and densities moving in a microchannel. The second is the Queralt *et al* research[4], studying a fluid behavior traveling across a microchannel with several obstacles to the walls, recreating a roughening. The simplest experiment after a microchannel without imperfection is to add one of them to the microchannel. We propose to study the dynamics of the front numerically using a phenomenological  $\phi^4$  model with a localized force  $F(x, y)$  to recreate the notch. The  $\phi^4$  equation has kink and antikink solutions that in the phase space represent heteroclinic trajectories joining minima points corresponding to different steady states that in the experiment, can be regarded as the air and fluid states.

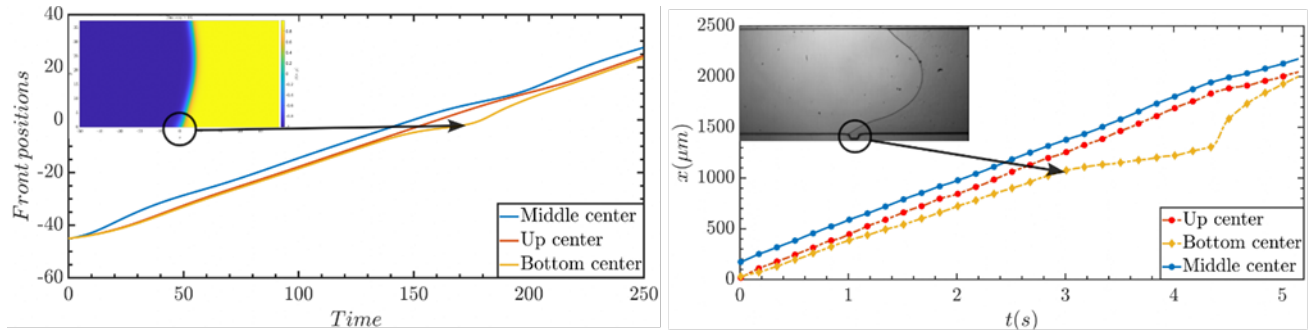


Figure 1: (a) Position as a function of time of three points of the front simulation, using a  $\phi^4$  model with a damping term  $\gamma(y) = \gamma_0 y^2$  and a localized force. (b) Experimental front with constant caudal  $Q_0 = 2 \mu\text{l/s}$  traveling through a microchannel with a notch. The simulation reproduces the behavior of the experimental front to reach the imperfection.

## Results and discussion

The simulation shows that a flat front presents damping oscillations after overcoming the imperfection. These oscillations depend on the front velocity reaching the notch. The velocity profile to the front with curvature mimics a laminar flow into a pipe. The front curvature removes the oscillations along the interface, being apparent at the front zone closer to the imperfection, matching with the experimental observations.

## References

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