Effect of an Uncertain Symmetry-Breaking Parameter on the Global Dynamics of the Duffing Oscillator

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Abstract. An adaptative phase-space discretization, based on an operator approach, is here employed to investigate the influence of a symmetry-breaking parameter on the global dynamics of the symmetric Duffing oscillator, in particular the basins' boundaries, attractors' distributions, and manifolds. The results highlight the importance of uncertainty analysis on the global structures of dynamical systems with competing attractors.

Introduction

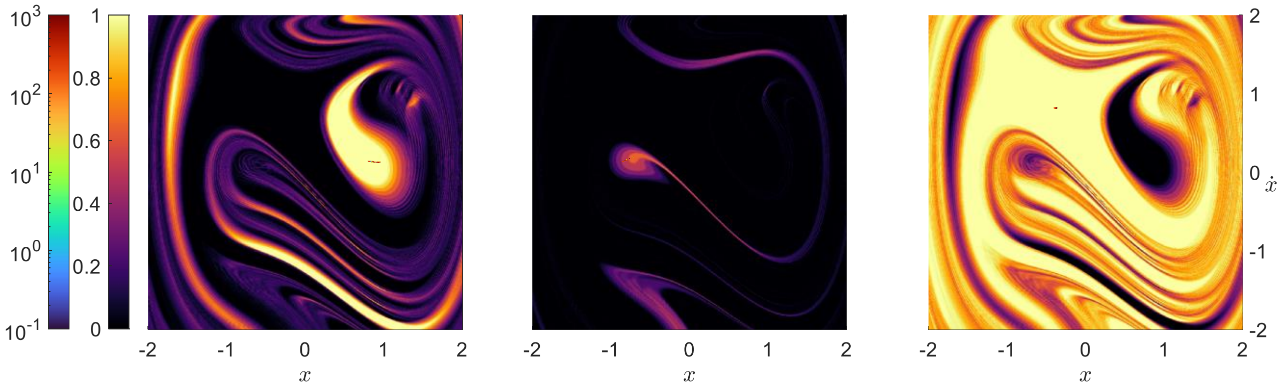
The symmetric Duffing oscillator is well studied in literature and may describe a plethora of events in sciences and engineering [1], including systems with one or two potential wells. However, in many applications ranging from quantum physics to engineering, a symmetry-breaking effect described by a quadratic nonlinear term is an important feature, and the magnitude of the quadratic term is often unknown. A well-known example is the nonlinear response of imperfect structures liable to stable or unstable symmetric buckling, such as plates and shells, where the magnitude of the geometric imperfections leading to asymmetry in the potential energy profile is unknown. Similar behavior is observed in the asymmetric Helmholtz-Duffing oscillator [2]. Here the influence of the uncertainty of the quadratic term coefficient, α, on the global dynamics of the following Duffing oscillator is investigated

$$\ddot{x} + 0.1\dot{x} - x - \alpha x^2 + x^3 = \lambda \sin(\omega t).$$

For $\alpha = 0$, the system exhibits a symmetric double-well potential function, whereas for $\alpha \neq 0$, the symmetry is broken.

Results and discussion

An adaptative phase-space discretization [3] is employed in the global analysis of the oscillator. It allows to refine complex basins' boundaries and then observe how the uncertainty in α affects the global dynamics. Depending on the forcing parameters (λ; ω), the system can present different numbers of competing solutions. For $\lambda = 0.35$, $\omega = 0.8$ three coexisting solutions are observed for the deterministic system: two in-well and one cross-well solutions. Assuming that α is uniformly distributed ($\alpha \sim U(0; 0.3)$), stochastic global structures, that is, attractors, basins, and manifolds distributions are obtained. The obtained basins are shown in Fig. 1. The left and right in-well attractors are deeply influenced by the symmetry-breaking, indicating a high sensitivity. Particularly the left in-well attractor, with its already small basin, loses stability.



(a) right well attractor (b) left well attractor (c) cross-well attractor

Figure 1: Attractors (left color bar) and basins (right color bar) probability distributions for $\lambda = 0.35$, $\omega = 0.8$, and $\alpha \sim U(0; 0.3)$.

References

[1] Kovacic, I., & Brennan, M. J. (2011). The Duffing equation: nonlinear oscillators and their behaviour. John Wiley & Sons.

[2] Lenci, S., & Rega, G. (2004). Global optimal control and system-dependent solutions in the hardening Helmholtz–Duffing oscillator. Chaos, Solitons & Fractals, 21(5), 1031-1046.

[3] Benedetti K C B 2022 Global Analysis of Stochastic Nonlinear Dynamical Systems: an Adaptative Phase-Space Discretization Strategy (Pontifical Catholic University of Rio de Janeiro, PUC-Rio) Rio de Janeiro, Brazil