# Differential Formulation of the Vaiana-Rosati Model

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**Abstract**. We present a differential formulation of a novel uniaxial rate-independent hysteretic model, denominated Vaiana-Rosati Model, that is capable of simulating complex generalized force-displacement hysteresis loops, ranging from the asymmetric, pinched, S-shaped, flag-shaped ones to those obtained by their arbitrary combination. Such a formulation drastically simplify the model implementation when the nonlinear equilibrium equations of a generic hysteretic mechanical system are expressed in the state-space form.

## Introduction

In the field of nonlinear dynamics, the second-order nonlinear Ordinary Differential Equations (ODEs), governing the complex response of hysteretic mechanical systems, are typically reformulated as a set of first-order nonlinear ODEs by introducing the state-space variables [1]. Consequently, hysteretic models are classically expressed in such a way that the output variable is evaluated by solving a differential equation that can be directly added to the set of first-order nonlinear ODEs to be numerically solved.
Conversely, more recent hysteretic models, such as the Vaiana-Rosati Model (VRM)[2], allow for the evaluation of the output variable by means of closed-form expressions having algebraic or transcendental nature.

## Vaiana-Rosati Model

The VRM offers a series of advantages over other hysteretic models available in the literature. Indeed, it: i) allows for the evaluation of the generalized force in closed form thus requiring a reduced computational effort, ii) is capable of reproducing different types of complex hysteresis loops, as illustrated in Figure 1, iii) is based on two set of eight parameters that allow one to manage, respectively, the loading and unloading phases, iv) adopts parameters describing specific theoretical and/or experimental properties of the hysteresis loops thus simplifying the identification procedure, and v) can be easily implemented in a computer program.

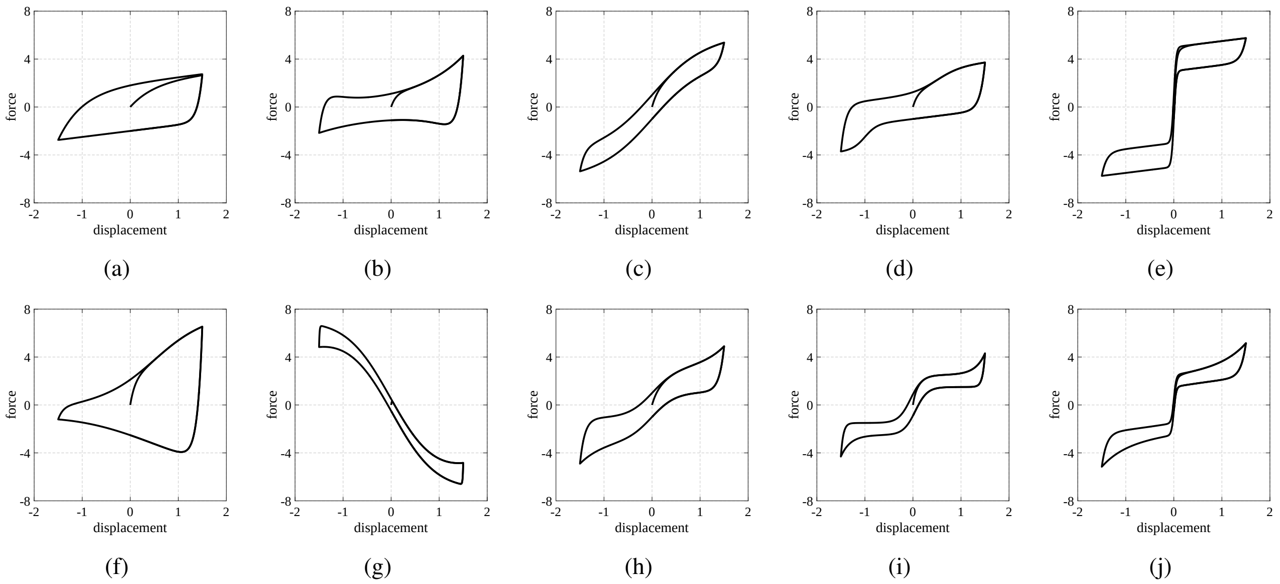


Figure 1: Some types of hysteresis loops simulated by the VRM.

## VRM Differential Formulation

In order to foster the use of the VRM in nonlinear dynamics, we present the related differential formulation for which the generalized force $f$, representing the model output variable, can be evaluated by solving the following first-order ODE:

$$\dot{f}(u, \dot{u}) = [a f(u) + b(u)]\, \dot{u}, \tag{1}$$

where the overdot denotes differentiation with respect to time $t$, $u$ ($\dot{u}$) represents the generalized displacement (velocity), $a$ is constant function, whereas $b(u)$ is a nonlinear function of $u$.

## References

[1] Formica G., Vaiana N., Rosati L., Lacarbonara W. (2021) Pathfollowing of High-Dimensional Hysteretic Systems Under Periodic Forcing. *Nonlinear Dyn.* **103**:3515–3528.

[2] Vaiana N., Rosati L. (2023) Classification and Unified Phenomenological Modeling of Complex Uniaxial Rate-Independent Hysteretic Responses. *Mech. Syst. Signal Pr.* **182**:109539.