Modelling Thermoelastic Damping in Nonlinear Plates with Internal Resonance

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Abstract. We use a nonlinear, reduced-order, coupled thermo-elastic model, derived from von Kármán plate equations to study the resonant response of simply supported thin rectangular plates under free and forced vibrations and understand the experimentally observed behaviour, specifically the change in temperature. The model is studied using Galerkin approximation, harmonic balance and numerical continuation techniques. Multimode response for plates is considered for non-resonant excitation as well as near 1:1 resonance between the second and third eigenmodes of the system, with harmonic forcing close to a natural frequency. Internal resonance and associated coupled-mode dynamics is observed, with noticeable decrease in modal amplitudes and some increase in plate natural frequencies due to thermo-elastic coupling.

Introduction

Experimental study of nonlinear vibration of 3D printed rectangular plates [1], fabricated using nonlinear materials, show significant increase in temperature due to continuous motion, suggesting strong coupling between mechanical motion and thermal response of the system. A thermo-mechanical constitutive model for thin plates, based on the Berger's approximation of von Kármán plate theory and energy equations [2], is used to study the vibration response of these under free and forced vibration conditions. A reduced set of coupled nonlinear ODE's for the evolution of transverse displacement $w(x, y, t)$ as well as temperature variables, the in plane stress ($N^T(x, y, t)$) and bending moment ($M^T(x, y, t)$), is obtained by the Galerkin projection of coupled thermal-structural PDEs onto the free vibration modes of the S-S-S-S plate [3]. One-mode and two-mode spatial approximations are considered. For example, for the one-mode approximation, letting $w = w_1(t)\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$, $M^T = m_1(t)\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$ and $N^T = n_1(t)\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$ gives the following set of coupled nonlinear ODEs:

$$\ddot{w}_1(t) + 0.05\dot{w}_1(t) + w_1(t) + 0.334w_1^3(t) - 0.067m_1(t) - 0.0014\, w_1(t)n_1(t) = Fcos(\Omega t) \quad (1)$$

$$\dot{n}_1(t) + 0.002n_1(t) + 7\times10^{-4}T_\infty + 0.0994w_1(t)\dot{w}_1(t) = 0 \quad (2)$$

$$\dot{m}_1(t) + 0.337m_1(t) + 1.18\dot{w}_1(t) = 0 \quad (3)$$

Nonlinear coupling is seen in the above, only for the w_1 and $n_1$equations. These equations are studied using direct numerical integration, through the harmonic balance method and by using pseudo arc-length based continuation schemes. They are also compared to finite element based results.

Results and Discussion

Single-mode approximation of the system shows significant damping being introduced by the thermo-elastic coupling, which affects both the amplitude and frequency of the system. Multimode response is studied for the case of a plate with1:1 relation between the second and third eigenmode frequencies, achieved through a precisely chosen aspect ratio of 1.633 for the plate [3]. Internal resonance is observed, resulting in activation of both modes while the external force is orthogonal to one of the modes. The coupling of thermal response again results in decrease in the amplitude, while increasing the resonant frequency. There is a corresponding increase in the temperature shown in Figure 1 at the midpoint of the plate. Thermal coupling also leads to an increase in the frequency range corresponding to the coupled mode.

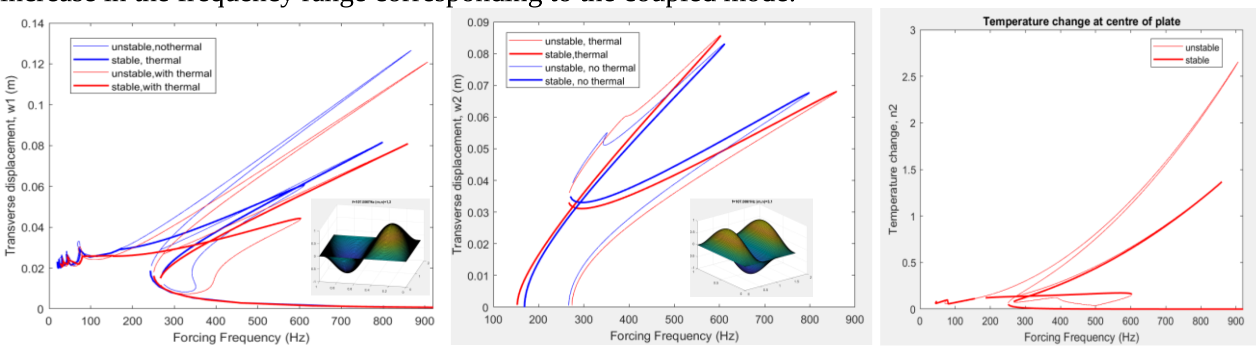

Figure 1. Multiple solution branches and bifurcations for: a) Mode 2 and b) Mode 3 transverse response for the two-mode model with 1:1 resonance; c) Temperature variation at the plate midpoint. Thick/thin lines corresponds to stable/unstable solutions respectively.

References

[1] Bilal, N., Tripathi, A., & Bajaj, A.K. (2020). On experiments in harmonically excited cantilever plates with 1:2 internal resonance. *Nonlinear Dyn 100, 15–32.*

[2] Saetta, E., & Rega, G. (2014). Unified 2D continuous and reduced order modeling of thermomechanically coupled laminated plate for nonlinear vibrations. *Meccanica* 49(8), 1723-1749.

[3] Chang, S. I., Bajaj, A.K., & Krousgrill, C.M. (1993). Non-Linear vibrations and chaos in harmonically excited rectangular plates with one-to-one internal resonance. *Nonlinear Dyn* 4, 433–460.