Stability Analysis of Large-Scale Multibody Problems using Lyapunov Exponents

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Abstract. Lyapunov Characteristic Exponents are stability indicators of solutions to nonlinear problems. However, their estimation can be very demanding, in terms of both complexity of implementation and sheer computational cost, limiting their applicability to large-scale multibody dynamics problems. This paper addresses the effective use of Jacobian-less methods to estimate Lyapunov Exponents from large-scale multibody problems.

Introduction

Lyapunov Characteristic Exponents (LCE) or, in short, Lyapunov Exponents (LE), represent the spectrum of the Cauchy problem $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t)$, $\mathbf{x}(t_0) = \mathbf{x}_0$. Considering a solution $\mathbf{x}(t)$, they are defined as

$$\lambda_i = \lim_{t\to+\infty} \frac{1}{t} \log \|{}_i\mathbf{x}(t)\|$$

where ${}_i\mathbf{x}(t)$ is the solution of the linear, time-variant (LTV) problem ${}_i\dot{\mathbf{x}} = \mathbf{f}_{/\mathbf{x}}(\mathbf{x}(t), t){}_i\mathbf{x}$, ${}_i\mathbf{x}(t_0) = {}_i\mathbf{x}_0$. As many LCEs as the size of the problem's state exist; however, estimating all of them is critical, as in theory independent values of ${}_i\mathbf{x}_0$ should be selected, such that each of them evolves along an independent direction. Numerically, even the faintest perturbation of each ${}_i\mathbf{x}_0$ with a contribution in the state subspace direction resulting in the largest λ_i, the maximum LCE (MLCE), would make all the limits converge to the MLCE itself. Numerical methods have been devised to overcome this issue; among them, the continuous QR and SVD methods, and the discrete QR method are the most effective. However, these methods require the Jacobian matrix of the problem, $\mathbf{f}_{/\mathbf{x}}(\mathbf{x}(t), t)$, evaluated along the reference trajectory $\mathbf{x}(t)$, and can hardly be formulated for problems described by Differential-Algebraic Equations (DAE), as is the case of the most popular multibody formulations. In this paper, we propose to use Jacobian-less methods to estimate LCEs of large-scale multibody dynamics problems directly from the time histories that result from the solution of the original Cauchy problem. One such method is proposed in [1].

Results

Complex multibody dynamics problems often occur in rotorcraft aeromechanics. Owing to the large linear and angular motion a helicopter rotor is subjected to, the kinematic complexity and variety of arrangements existing (and foreseen) for rotor hubs, and the highly nonlinear constitutive properties of several components, like lead-lag dampers, multibody dynamics is ideal for their analysis. Typical analyses may result in (nearly) time-periodic solutions; for this reason, a typical approach to stability analysis of periodic orbits is the Floquet-Lyapunov method; however, it may be convenient to devise a method that does not require the computation of a periodic orbit, or even its mere existence. LCEs are ideal, in this case, as stability analysis can progress along with the numerical integration of the problem.

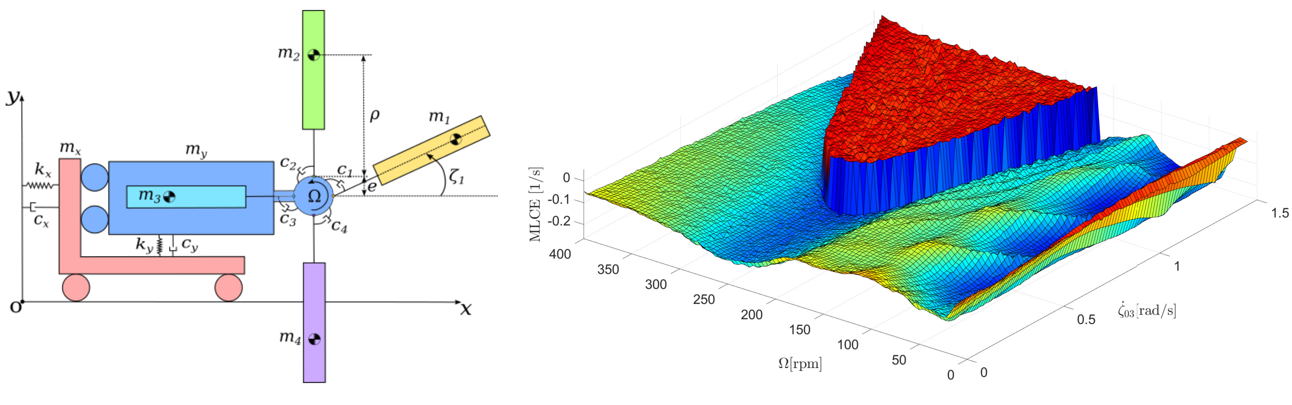

Figure 1: Left: rotor/support model; right: sensitivity of MLCE to amplitude of initial perturbation.

In Fig. 1 we present a sketch of a popular rotorcraft ground resonance model and the MLCE estimated using the method proposed in [1] as a function of the rotor angular velocity, Ω, and the amplitude of the initial perturbation of the angular velocity of a blade. The method will be assessed by comparing results with equivalent ones, when possible, and its application to more complex problems will be illustrated.

References

[1] Rosenstein M. T., Collins J. J., De Luca C. J. (1993) A practical method for calculating largest Lyapunov exponents from small data sets. *Physica D: Nonlinear Phenomena*, **65**(1–2):117–134 doi:10.1016/0167-2789(93)90009-P