Reconstructing Nonlinear Backbone Curves from Smooth Coordinate Decomposition of Multivariate Impulse Response

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Abstract. Nonlinear modal coordinate pairs, obtained from the smooth coordinate decomposition of the impulse response trajectories, are used to estimate nonlinear frequency-amplitude backbone curves. This method allows simultaneous estimation of multiple backbone curves for all excited modes.

Introduction

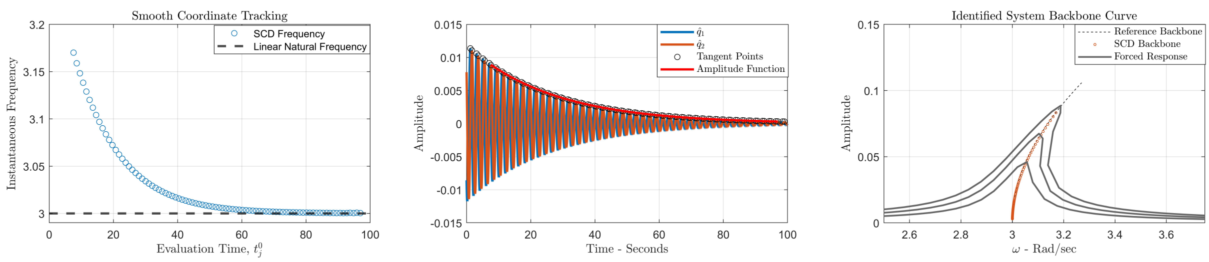
A system's modal parameters' identification is an essential step during the design of engineering structures. Recent advancements in innovative lightweight structures and novel materials require engineers to analyze systems that are inherently nonlinear. Thus, practitioners must find a suitable method to characterize nonlinear phenomena. The backbone curves provide a description of the frequency-amplitude relationship for the undamped and unforced response of the system and are instrumental in the study of the nonlinear system response. Recent experimental nonlinear system identification efforts have focused on obtaining these curves from the output response data [1, 2]. Here, we propose using a state space formulation of the Smooth Coordinate Decomposition (SCD) [3] to estimate the amplitude-frequency-dependent behavior embedded within the smooth orthogonal coordinates.

Results and Discussion

Consider the system response in the form of a trajectory matrix, $Y = [X\ \hat{X}] \in \mathbb{R}^{m\times 2n}$ where $\hat{X}$ is a temporally correlated version of X (e.g., state velocities). The SCD looks for a basis, Φ, that maximizes the variances of the projected field while minimizing their roughness. That is, we are looking for projections of the system response onto the smoothest (i.e., slowest decaying) modes (SMs), Φ. The resulting smooth coordinates (SCs) will have the following structure,

$$Q = Y\Phi^{-T} = \begin{bmatrix} q_1(t_1) & \hat{q}_1(t_1) & q_2(t_1) & \hat{q}_2(t_1) & . & . & q_n(t_1) & \hat{q}_n(t_1) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ q_1(t_n) & \hat{q}_1(t_n) & q_2(t_n) & \hat{q}_2(t_n) & . & . & q_n(t_n) & \hat{q}_n(t_n) \end{bmatrix}. \tag{1}$$

where the columns of Q will come in distinct pairs, $q_i(t)$ and $\hat{q}_i(t)$, that have the same smoothness but will be 90° phase shifted from each other due to the orthogonality condition. The SC pairs embed the frequency-amplitude relationship for each single-mode response (assuming no internal resonances) of a nonlinear multi-degree-of-freedom arbitrarily excited system. Furthermore, the orthogonality and equivalent oscillation pattern within the pairs allow one to utilize the coordinates in the same way as the analytical signal in the Hilbert Transform-based approach. Hence, the mode separation and construction of a suitable pseudo-analytical signal representation are done within one step. One can then assemble two sequences that pair the frequencies, $\omega_n(t_j)$, and amplitudes, $A(t_j)$, for each evaluation time, t_j, within each mode.



References

[1] Londono J. M., Cooper J. E., and Neild S. A. "Identification of systems containing nonlinear stiffnesses using backbone curves," Mechanical Systems and Signal Processing, vol. 84, pp. 116–135, 2017.

[2] Feldman M., "Hilbert transform applications in mechanical vibration," John Wiley & Sons, 2011.

[3] Chelidze D. and Zhou W., "Smooth orthogonal decomposition-based vibration mode identification," J. of Sound and Vibration, vol. 292(3), pp. 461–473, 2006.