Fractional damping term in the Helmholtz and Duffing nonlinear oscillators.

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Abstract We present the fractional derivative impact on the dynamics of two systems. In both cases we focus on the cooperation of the damping term, the fractional derivative parameter and the external forcing to study it in the underdamped and the overdamped regime.

Introduction

We analyse the dynamics of the nonlinear Helmholtz and the Duffing oscillator with a fractional order damping. We have investigated the effect of taking the fractional derivative in the dissipative term in function of the parameter α. In both system we have studied the time to reach the asymptotic behaviours, being leave the well or reach an attractor, in function of the fractional parameter. The simulations have been carried for the underdamped and overdamped case.

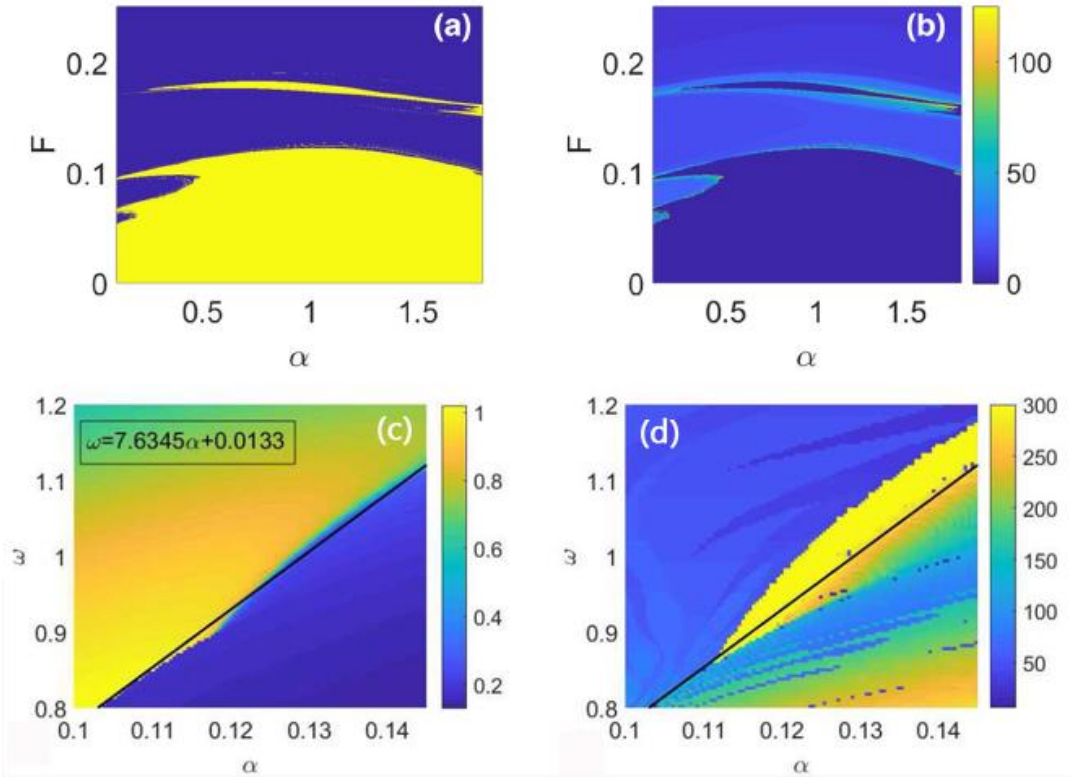

Figure 1: Figures (a) and (c) show the final state for the overdamped regime. Figure (b) and (d) show the time for the system to reach the final state.

Results and discussion

Among other results, we show that in the fractional case, in both systems, the overdamped regime, that in the non-fractional case is easily predictable, becomes more interesting and unpredictable. Moreover, we discuss how some important behaviours of the systems change in function of the fractional parameter, see as an example Figs. 1. The figures show for the overdamped regime the final state of the systems (on the left, figures (a) and (c)) and the times to reach it (on the right, figures(b) and (d)). On the top, for the Helmholtz oscillator, the final state is the escape or not from the well. On the bottom, for the Duffing oscillator, the final state is the amplitude of the oscillations. Moreover, in the Duffing oscillator a resonance-like phenomenon induced by the fractional term can be found. To summarize, the results show that the overdamped case with fractional derivative becomes more interesting than in the classical case.

References

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