Dynamics of an intermittent HIV treatment using piecewise smooth vector fields with two switching manifolds

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Abstract. In this paper we study the dynamics of a piecewise smooth vector field modeling an HIV treatment where the patient is recurrently submitted and removed from drugs administration. In fact, the protocol says that the drugs are administrated when the level of CD4$^+$ T defense cells is smaller than a fixed number C^T_{off}. When the level of CD4$^+$T cells is greater than a fixed number C^T_{on} (distinct from C^T_{off}). Moreover, the orbits of the piecewise smooth vector fields are trapped within a compact set, which proves that the protocol controls the disease.

Introduction

Differential equations are widely used to simulate the kinetics of viral infections and their therapies. Basically, viral dynamics models consist in systems of ODEs that describe the interactions between infected cells, susceptible and the infectious form of a virus that generally behaves differently in patients with or without treatment, makes the mathematical modeling more realistic when modeled by piecewise smooth vector fields (PSVFs).
The HIV is a retrovirus that attacks the defense cells of the infected causing enormous vulnerability in its host and has antiretroviral therapy (RT) as an alternative to control the disease. Usual RT is based on two classes of antiretroviral drugs known as reverse transcriptase inhibitors (RTIs) and protease inhibitors (PIs). Basically, these drugs are responsible for impairing the mechanisms that retroviruses use for their proliferation.
In [1] were used ODEs to simulate the monitoring of the number of defense cells in patients carrying HIV submitted to an intermittent treatment. More precisely, a threshold value C^T was considered at which antiretroviral therapy is started (or stopped) when the cell count $CD4^+T$ is below (or above) C^T. For more details, see [1].
Although the cited paper has significant results, in practice treatments with antiretrovirals do not occur exactly in this way. A more realistic simulation would be instead of just a single threshold value to activate (or deactivate) the equations of the PSVF (see Figure 1). In this scenario, the treatment starts when the cell count is below a threshold C^T_{on} and stops when the cell count exceeds the other threshold C^T_{off}, and the treatment is kept until the next one limit is exceeded.

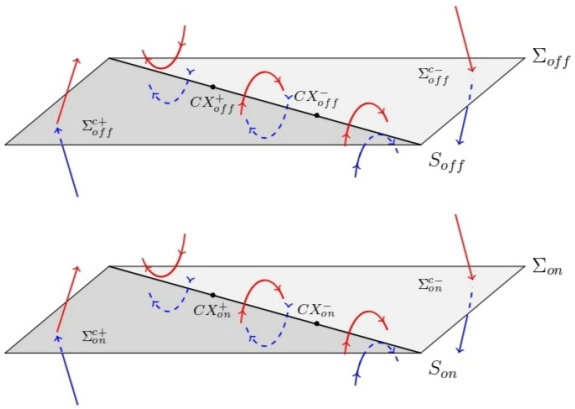

Figure 1: Dynamics of $X^{\pm}$ in Σ_{on} and Σ_{off}.

Results and discussion

In this paper, we study a PSVF that addresses the behavior we have just described in which when establishing conditions for the limits C^T_{off} and C^T_{on} we observe that the solutions are constrained in a compact set containing a closed orbit. This result, from a medical point of view, is extremely relevant, as it proves the efficiency of the treatment in controlling the disease.

References

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