Effects of controller-induced dynamics on experimental bifurcation analysis

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Abstract. Control-based continuation (CBC) is an experimental method that uses feedback control to probe the dynamics of a physical system. CBC relies on finding control targets that render the control system non-invasive, recovering the naturally occurring system behaviours without modifying their geometry or location. Here, we highlight a case where a CBC experiment fails due to the controller inducing spurious, invasive dynamics in the system. A limit cycle in a slow-fast system is controlled successfully, however the controller causes it to collapse through a canard explosion in the prediction step of a continuation. We propose strategies to overcome this issue, and use it as a case-study to highlight how control strategies and residual invasiveness can impact the results of an experimental bifurcation analysis.

Introduction

A feedback controller is noninvasive if it stabilises a control target $x^*(t)$, and the associated control action $u(x^*, x, t) \equiv 0$. Controllers can nevertheless exert a nonzero control action when pushing a system onto a target, so that unstable responses become stabilised and directly observable [1]. Zero control action guarantees that observed features are intrinsic to the system, as the controller will not modify the dynamics. In practise however, small invasiveness is to be expected, for example from feedback delays in a controller, or through intentional perturbations and random errors when solving for noninvasive targets. For some control schemes, this invasiveness can cause the controller to modify the system dynamics in unwanted ways. Continuity suggests that small invasive errors in x^* lead to small control actions $u(x^*, x, t)$, and hence small errors between intrinsic and observed system responses. Here we propose a counterexample, where a small error in a control target causes the controller to induce a catastrophic error in the system response. Our example arises in the CBC of relaxation oscillations. Large-amplitude oscillations in a slow-fast system can rapidly collapse through a canard explosion. Due to the autonomous nature of our chosen control law, a canard explosion is able to happen in the prediction step of a continuation, causing CBC to fail.

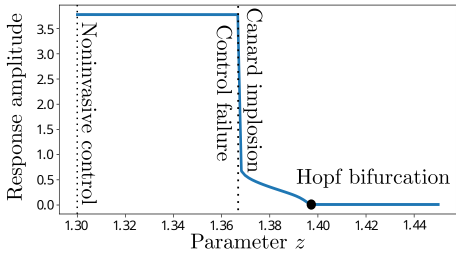

Figure 1: Angle-encoded controller [3] causes the dynamics of interest to collapse through a canard explosion as z is increased.

Results

We consider as a case study the Fitzhugh-Nagumo model [2]. An autonomous control scheme is constructed using the method of angle-encoding [3]. This proceeds by embedding time-dependent observations $x(t)$ in a reconstructed limit cycle, and calculating the angle ϕ of an embedded state relative to a reference direction and origin. Proportional control is applied with errors defined between measurements $x(\phi)$ and an angle-indexed control target $x^*(\phi)$, to phase-lock control targets to system responses. This control strategy acts as a phase constraint, producing a locally unique oscillatory solution to the noninvasive equations, and avoids the need to compute an oscillatory period. The control scheme is initialised with a noninvasive control target from open-loop data, then system bifurcation parameter z is increased without changing the control target, to emulate natural parameter continuation. As illustrated in Fig. 1, the controlled limit cycle collapses through a canard explosion, destroying the dynamics of interest. This happens due to a combination of the slow-fast dynamics, and the autonomous control scheme. Natural parameter continuation modifies the system parameter in a prediction step, whereas pseudo-arclength continuation also changes the control target; this causes the canard explosion to happen sooner. We use observations about the control law to justify how this issue can be avoided, by moving the angle-defining polar origin. More generally, we emphasise that care must be taken to ensure that CBC experiments remain within a parameter range for which intrinsic dynamics can be observed, and that this parameter range may be small.

References

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