Regular and complex behaviour in the pendulum system under a magnetic field.

Yuliia E. Surhanova* and Yuri V. Mikhlin*

Department of Applied Mathematics, National Technical University "Kharkiv Polytechnic Institute", Kharkiv, Ukraine

Abstract. Dynamics of two coupled pendulums under a magnetic field are considered by taking into account a dissipation in the system. Inertial components of the pendulums are essentially different, and a ratio of masses is chosen as a small parameter. Padé approximants is used for the magnetic forces approximation. The method of multiple scales is used to construct nonlinear normal modes (NNMs), one of them is a localized one. An appearance of the complex behaviour when the system parameters change is investigated.

Introduction

The system containing two pendulums under the influence of electromagnetic force is studied in a few papers [1,2]. Here we consider a such dissipated system of two pendulums under a magnetic field when the inertial characteristics of these pendulums are essentially different. To describe the system dynamics, first of all, the nonlinear normal modes (NNMs) [3,4], one of them is a localized one, are constructed by the multiple scales method. The used Padé approximants demonstrate a good correspondence of the magnetic forces analytical presentation with experimental results presented in [1,2].

Equations of motion of the system under consideration concerning the pendulum angles are the following:

$$\begin{cases} \varepsilon I\ddot{\varphi}_1 = \varepsilon M_{mag_1} + \varepsilon M_{D1}(\varphi_1,\varphi_2) + \varepsilon M^{(g)}(\varphi_1) + M^{(k)}(\varphi_1,\varphi_2), \\ I\ddot{\varphi}_2 = \varepsilon M_{mag_2} + \varepsilon M_{D2}(\varphi_1,\varphi_2) + M^{(g)}(\varphi_1) + M^{(k)}(\varphi_1,\varphi_2). \end{cases} \tag{1}$$

Here I is the moment of inertia, $M_{mag_{1,2}}$ represent magnetic forces, $M_{D1,2}$ are damping moments, $M_{D1} = -C_1 - C_e(\varphi_1 - \varphi_2)$, $C_1\dot{\varphi}_1$ and $C_2\dot{\varphi}_2$ are moments of resistance to viscous air, $C_e(\dot{\varphi}_1 - \dot{\varphi}_2)$ и $C_e(\dot{\varphi}_2 - \dot{\varphi}_1)$ are damping moments created by the elastic element, $M^{(g)}, M^{(k)}$ are returning moments of gravity and elastic forces, respectively, $M^{(g)} = -mgs\sin\varphi_{1,2}\,, M^{(k)} = -k_l(\varphi_1 - \varphi_2), r = mgs,$ instead of the sine function, we use the shortened Taylor series, $\sin(\varphi) \cong \varphi - \frac{\varepsilon}{6}\cdot\varphi^3$, s is a distance between the pendulum center of mass and the axis of rotation, m is the biggest pendulum mass, ε is a small parameter characterizing the ratio of two pendulums masses, k_l is the binding elastic element stiffness.

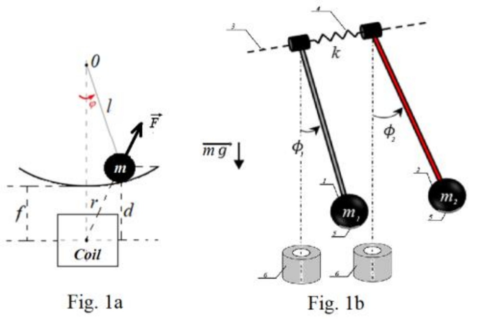

Figure 1: Schemes of the single particle and the magnetic force interaction (Fig.1a) and of the coupled pendulums system (Fig.1b).

Results and Discussion

The multiple scales method is used. In zero approximation, the relation $\varphi_{10} = \varphi_{20}$ is obtained for the coupled (in-phase) vibration mode, and the relation $\varphi_{20} = 0$ is obtained for the localized vibration mode. Numerical simulations show a good exactness of the obtained analytical solution for relatively small values of the parameter ε. With increasing the parameter ε there is a gradual transition from the localized vibration mode to the out–of–phase one. Numerical simulation, including calculation of the frequency spectrum, and analysis of this NNMs stability under change of the system parameters and initial angles, permits to reveal of regular behavior in the shape of NNM, as well irregular behavior, preferably for small values of the initial angles of the pendulums, and for relatively small values of the parameters k_l and s.

References

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