

# Stability and instability of a complex biodynamical discrete structure on a cantilever coupled to nonlinear springs

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**Abstract.** To withstand the rigors of wind, rain, and their own weight, some young seedlings need assistance in the first year after planting at the permanent place in the form of single, double or triple staking methods. Using a previously developed model of complex discrete cantilever coupled with nonlinear spring we investigate the stability and instability of the complex structure regarding the number of nonlinear springs to which the structure is coupled. The geometric nonlinearity of the system is introduced by a spring with cubic nonlinear properties that oscillates in the horizontal plane. Stability of oscillations of these complex structures is important for proper stacking of young seedlings with different canopy shapes. First asymptotic approximations of nonlinear differential equations along amplitudes and phases of nonlinear modes of three-frequency oscillations of considered complex structure is derived. An analysis of interactions between nonlinear modes is done.

## Introduction

It is of a great interest that staking of the young seedlings is done properly ensuring oscillations of the trunk that will stimulate the root growing and proper anchoring to the ground [1,2]. Stability of forced oscillations of double and triple staking methods is analysed using a modification of a previously developed model [3] of a young seedling staked with one stake. The purpose of the study is to analyse which canopy shapes are the most suitable for each particular staking method.

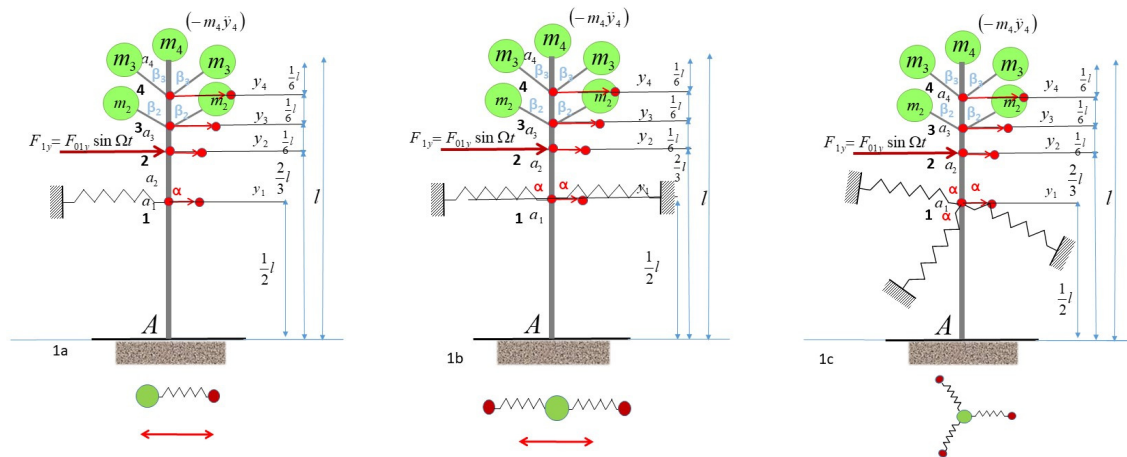


Figure 1: Complex discrete structure on a cantilever with: 1a. one spring with nonlinear properties 1b. two springs spring with nonlinear properties 1c. tree springs with nonlinear properties.

A young seedling staked with single, double or triple staking methods (Fig.1) is modelled as complex discrete biodynamical structure on a cantilever coupled with one/two/three nonlinear elastic springs at 2/3 of cantilever (tree stem) height. Springs with cubic nonlinear properties oscillate in the horizontal plane. Stability and instability of forced three-frequency nonlinear oscillations of these complex system under an external three-frequency force is analyzed.

## Results and discussion

First asymptotic approximations of nonlinear differential equations along amplitudes and phases of nonlinear modes of three-frequency oscillations of considered complex discrete biodynamical structure are derived. An analysis of the interactions between nonlinear modes is done. Stability of structures on is analyzed by graphs of characteristic equations for different geometric parameters of the model.

## References

- [1] Ramanujam L. A Nonlinear Model for Wind-Induced Oscillations of Trees. (2012). <http://scholarworks.umass.edu/theses/943/>.
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- [3] Hedrih (Stevanović) K., Hedrih A. (2022) Mathematical modelling of nonlinear oscillations of a biodynamical system in the form of a complex cantilever. *App. Math. Modell.* VSI: DSTA 2021, 112:110-135.