New existence results for sequential generalized nonlinear Hilfer fractional q-differential inclusions via multi-point boundary conditions

Mohammad Esmael Samei[*], Ahmad Ahmadi[*] and Adel Lachouri[**]

[*]*Department of Mathematics, Faculty of Basic Science, Bu-Ali Sina University, Hamedan, Iran, ORCID # 0000-0002-5450-3127*

[**]*Laboratory of Applied Mathematics, Annaba University, Annaba, Algeria, ORCID # 0000-0002-6269-8833*

Abstract. In this paper, we talk about the presence the existence of solutions for a generalized Hilfer fractional q-differential inclusions dynamical system with multi-point boundary conditions. Both cases of convex and nonconvex esteemed right-hand side are considered. Our gotten comes about are later within the outline of a generalized Hilfer fractional q-derivative with multi-point boundary conditions via the fixed point theorems for set-valued mappings. A few relevant illustrations illustrating the viability of the hypothetical comes about are displayed.

Introduction

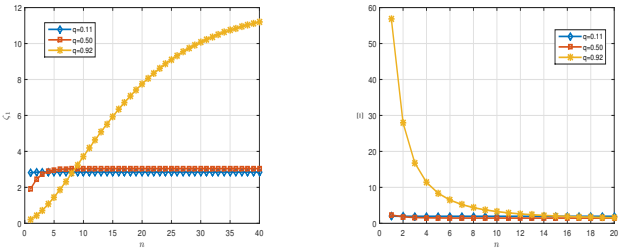
In the literature, there are diverse definitions of fractional derivatives and integrals e.g. Riemann-Liouville, Liouville-Caputo, Hadamard, Hilfer, etc [4]. Also, new some operators have been emerged to combine a broad category of fractional derivatives like ξ-Liouville-Caputo and ξ-Hilfer [7]. Recently, Asawasamrit *et al.* [1] have started the study of Hilfer fractional differential equation $(\mathbb{FDE})$ of the type ${}^{\mathrm{H}}\mathbb{D}_{\mathfrak{a}+}^{\tau_1,\tau_2}\varpi(\mathfrak{z}) = \mathfrak{p}(\mathfrak{z},\varpi(\mathfrak{z}))$, for $\mathfrak{z} \in \beth := [\mathfrak{a},\mathfrak{b}]$, with nonlocal integral boundary conditions $\varpi(\mathfrak{a}) = 0$, $\varpi(\mathfrak{b}) = \sum_{i=1}^{m}\eta_i\,\mathbb{I}_{\mathfrak{a}+}^{\sigma_i}\varpi(\delta_i)$, $\delta_i \in \beth$. Authors in [5] have investigated the existence and stability results of implicit problem for the fractional differential inclusions ($\mathbb{FDI}$s) involving ψ-Hilfer fractional derivative. In [6] the authors proved the existence uniqueness of solutions to the following problem involving generalized Hilfer (ξ-Hilfer) $\mathbb{FDI}$, $({}^{\mathrm{H}}\mathbb{D}_{\mathfrak{a}+}^{\tau_1,\tau_2;\xi} - k\,{}^{\mathrm{H}}\mathbb{D}_{\mathfrak{a}+}^{\tau_1-1,\tau_2;\xi})\varpi(\mathfrak{z}) = \mathfrak{p}(\mathfrak{z},\varpi(\mathfrak{z}))$ for $\mathfrak{z} \in \beth$, with multi-point boundary conditions $\varpi(\mathfrak{a}) = 0$, $\varpi(\mathfrak{b}) = \sum_{i=1}^{m}\eta_i\varpi(\delta_i)$.

Results and discussion

Motivated by the aforementioned works and inspired by [2, 3, 7], we prove the existence of solutions for a nonlinear q-fractional differential inclusions (q-$\mathbb{FDI}$s) involving left-sided ξ-Hilfer fractional q-derivative

$$\left({}^{\mathrm{H}}\mathbb{D}_{q;\mathfrak{a}+}^{\tau_1,\tau_2;\xi} - k\,{}^{\mathrm{H}}\mathbb{D}_{q;\mathfrak{a}+}^{\tau_1-1,\tau_2;\xi}\right)\varpi(\mathfrak{z}) \in \mathfrak{P}(\mathfrak{z},\varpi(\mathfrak{z})), \qquad \mathfrak{z} \in \beth, q \in (0,1), \tag{1}$$

with multi-point boundary conditions $\varpi(\mathfrak{a}) = 0$, $\varpi(\mathfrak{b}) = \sum_{i=1}^{m}\eta_i\varpi(\delta_i)$, where $\mathfrak{P} : \beth \times \mathbb{R} \to \mathcal{P}(\mathbb{R})$ is a set-valued map from $\beth \times \mathbb{R}$ to the family of $\mathcal{P}(\mathbb{R}) \subset \mathbb{R}$. We obtain existence results for the right-sided ξ-Hilfer inclusion problem (1) involving convex, nonconvex set-valued maps via some fixed point theorems. Observe that our problem (1) for different values of τ_2 and ξ is combining a wide class of BVP for q-$\mathbb{FDI}$s.



(a) U-H Stability (b) U-H-R Stability

Figure 1: Graphical representation of Ξ for q-$\mathbb{FDI}$ whenever $q \in \{0.11, 0.5, 0.92\}$ in Example 1.

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