A novel alternative formalism of the Wiener path integral technique - circumventing the Markovian assumption for the system response process

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Abstract. The formulation of the Wiener path integral (WPI) technique for determining the stochastic response of diverse dynamical systems has been developed to-date in conjunction with the Markovian assumption for the system response process. Herein, a novel WPI formalism is developed to account, in a direct manner, also for systems with non-Markovian response processes. In this regard, nonlinear systems with a history-dependent state, such as hysteretic structures or oscillators endowed with fractional derivative elements, can be treated in a straightforward manner. That is, without resorting to any ad hoc modifications of the WPI technique pertaining, typically, to employing additional auxiliary filter equations and state variables.

Introduction

Various methodologies have been developed over the last six decades in the field of stochastic engineering dynamics for determining response statistics of diverse structural and mechanical systems [1]. Indicatively, relying on the Markovian assumption for the system response process, a wide range of techniques have been developed for solving the Fokker-Planck partial differential equation governing the system response joint transition probability density function (PDF); see [2] for a broad perspective. Nevertheless, for a wide range of systems the convenient Markovian response assumption cannot be reasonably justified. Indicative examples include systems exhibiting hysteresis or subjected to non-white stochastic excitations. This challenge is bypassed, typically, by considering additional auxiliary filter equations and state variables. However, this kind of solution treatment relates usually to increased computational cost due to the increased dimensionality of the problem.

Results and discussion

Kougioumtzoglou and co-workers have developed recently a technique based on the concept of Wiener path integral (WPI) for stochastic response determination of diverse dynamical systems (e.g., [3-5]). Remarkably, the technique exhibits both high accuracy [4] and low computational cost [5]. However, the formulation of the WPI technique has been developed to-date in conjunction with the Markovian assumption for the system response process. In this paper, an alternative novel formalism is developed that circumvents the Markovian response assumption. Specifically, considering the probability of a path corresponding to the Wiener (excitation) process, and employing a functional change of variables in conjunction with the governing stochastic differential equation, yields the probability of a path corresponding to the response process. This leads to representing the system response joint transition PDF as a functional integral over the space of possible paths connecting the initial and final states of the response vector. Overall, the veracity and mathematical legitimacy of the WPI technique to treat also non-Markovian system response processes are demonstrated. Illustrative numerical examples relate to nonlinear oscillators exhibiting hysteresis and endowed with fractional derivative elements. Comparisons with pertinent Monte Carlo simulation (MCS) data demonstrate the accuracy of the developed formalism (Fig.1).

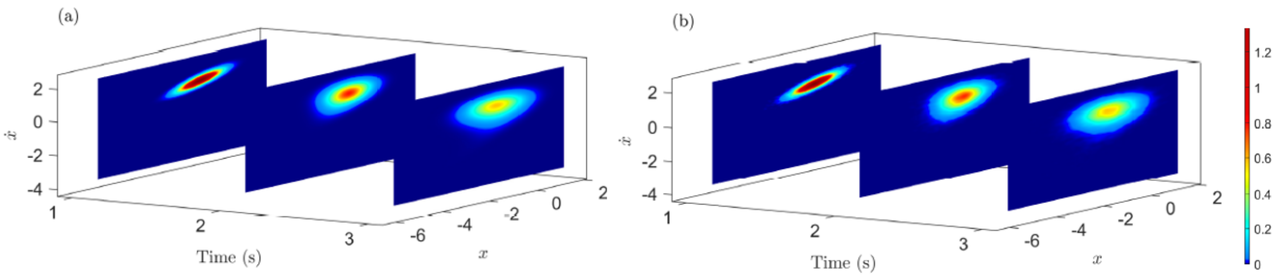

Figure 1: Non-stationary response joint PDF at indicative time instants corresponding to a stochastically excited oscillator with asymmetric nonlinearities and fractional derivative elements:(a) results obtained by the WPI technique; (b) comparison with MCS data.

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