Regulating dynamics through intermittent interactions

Shiva Dixit, Manaoj Aravind and Punit Parmananda
Department of Physics, Indian Institute of Technology Bombay, Powai, Mumbai 400 076*, India.*

Abstract. We demonstrate an efficient scheme to regulate the behaviour of coupled nonlinear oscillators through dynamic control of their interaction. It is observed that introducing intermittency in the interaction term as a function of time or the system state, predictably alters the dynamics of the constituent oscillators. Choosing the nature of the interaction - attractive or repulsive, allows for either suppression of oscillations or stimulation of activity. Two parameters Δ and τ, that reign the extent of interaction among subsystems are introduced. They serve as a harness to access the entire range of possible behaviours from fixed points to chaos. For fixed values of system parameters and coupling strength, changing Δ and τ offers fine control over the dynamics of coupled subsystems.

Introduction

Dynamics ranging from fixed points to chaos exist in nature [1] and mechanisms to regulate such dynamics are crucial for practical applications. Coupled nonlinear oscillators serve as a prototype to model many real-world systems [2] examples include the climate [3], population dynamics in ecosystems and financial markets . Consider $N(=2)$ identical m-dimensional nonlinear oscillators coupled via dynamic interaction in the following two ways:

$$\dot{\mathbf{X}}_i = F(\mathbf{X}_i) + \epsilon\gamma \sum_{j=1}^{N} (H\mathbf{X}_\mathbf{j} - \mathbf{X}_i), \tag{1}$$

where, $i, j = 1, 2$, and $i \neq j$. $\mathbf{X}_i$ represents state variables of the m-dimensional i-th oscillator and $F : \mathbb{R}^m \rightarrow \mathbb{R}^m$ is the vector field describing its intrinsic dynamics. $H(\mathbf{X}_i, t)$ takes the values 0 or 1 either as a function of time t or the state variables $\mathbf{X}_i$. When $H = 1$ the two oscillators are coupled to each other and when $H = 0$ they are completely isolated from each others' influence. When H depends on the state variables of the two oscillators, i.e.,

$$H(\mathbf{X}_i, t) = \begin{cases} 1 & \text{if } \mathbf{X}_i \in R' \\ 0 & \text{if } \mathbf{X}_i \notin R' \end{cases} \tag{2}$$

this yields a *state dependant interaction*. Where R' is a subset of the state space $\mathbb{R}^m$ where the interaction is active. A measure of this subset is obtained from the normalized fraction $\Delta = \Delta'/\Delta_a$, where Δ_a is the width of the uncoupled attractor along the direction of the coupled state variable and Δ' is the width of the region in which the coupling is active. *Time dependant interaction* is when H is explicitly dependant on time. For instance H can be a periodic step function of time period T as follows,

$$H(\mathbf{X}_i, t) = \begin{cases} 1 & \text{if } 0 < t \leq \tau' \\ 0 & \text{if } \tau' < t \leq T \end{cases} \tag{3}$$

Here, $\tau = \tau'/T$ is a measure for the fraction of time the interaction is active. These two parameters Δ and τ that control the degree of interaction between the coupled systems, allow us to harness a given coupled oscillator into desired stable states.

References

[1] M. Lakshmanan and K. Murali, Chaos in nonlinear os- cillators: controlling and synchronization, Vol. 13 (World scientific, 1996).
[2] A. T. Winfree, The geometry of biological time, Vol. 2 (Springer, 1980).
[3] M. Scheffer, Critical transitions in nature and society (Princeton University Press, 2020).

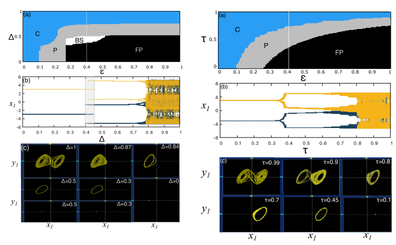

Figure 1: Suppression through space-dependant interaction(Left) Suppression through time-dependant interaction:(Right): (a) Different dynamical states of two coupled Chua oscillators in the parameter plane ($\epsilon - \Delta$). The regimes marked C, P, FP, and BS represent chaotic, periodic, fixed point and bistable (co-existence of oscillatory and fixed point state) state respectively. (b) Bifurcation diagram of the coupled Chua system (c.f. Eqs. 1) is plotted with interaction active state space Δ at $\epsilon = 0.41$, obtained by sampling the relative maxima and minima of the time history of $x_1(t)$.(c) Experimental phase portraits for $\epsilon = 0.41$ corresponding to various values of the control parameter Δ.