Dynamics of Purcell’s three-link microswimmer model with actuated-elastic joints

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Abstract. Purcell’s planar three-link microswimmer is a classic model of swimming at low-Reynolds-number fluid, inspired by motion of flagellated microorganisms. Many works analyzed this model, assuming that the two joint angles are directly prescribed in phase-shifted periodic inputs. In this work, we study a more realistic scenario by considering an extension of this model which accounts for joints’ elasticity and mechanical actuation of periodic torques, so that the joint angles are dynamically evolving. Numerical analysis of the swimmer’s dynamics reveals multiplicity of periodic solutions, depending on parameters of the inputs – frequency and amplitude of excitation, as well as joints’ stiffness. We numerically study bifurcations, stability transitions, and symmetry breaking of the periodic solutions, demonstrating that this simple model displays rich nonlinear dynamic behavior with actuated-elastic joints.

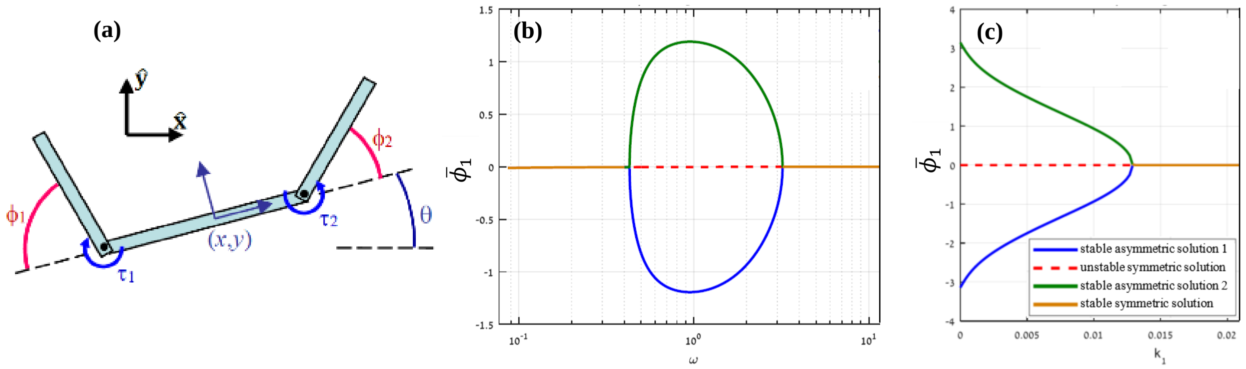
Introduction

Purcell’s three-link microswimmer model has been introduced in [1]. The model consists of three rigid links connected by two joints whose angles ϕ_1, ϕ_2 are assumed to be controlled as phase-shifted time-periodic inputs. The dynamic equations of motion of Purcell’s swimmer have been formulated in [2], assuming slender links under low-Reynolds-number hydrodynamics, and further analyzed in many other works. The work [3] introduced a modified model where one joint angle is periodically actuated as $\phi_1(t) = \varepsilon \sin(\omega t)$ while the other joint is *passive* and acted by a *torsion spring* with linear stiffness, so that the joint torque is $\tau_2 = -k\phi_2$. The work [3] considered the case of small amplitude $\varepsilon \ll 1$, where the only periodic solution of the system, which is orbitally stable, occurs with the passive joint angle $\phi_2(t)$ oscillating symmetrically about zero. Using asymptotic expansion, the swimmer’s response as a function of the input frequency ω was studied in [3], showing existence of optimal frequency that maximizes the net displacement per cycle.

In this work, we consider an extension of this model in [3], with actuated-elastic joints, whose torques are given as τ_i=$\varepsilon_i \sin(\omega t$+$\gamma_i)$-$k_i\phi_i$. This input can also be written in equivalent form of a local feedback law as $\tau_i = -k_i\left(\phi_i - \psi_i(t)\right)$ where $\psi_i(t) = \tilde{\varepsilon}_i \sin(\omega t$+$\gamma_i)$ is a desired reference angle to be tracked.

Results and discussion

Using numerical integration of this dynamical system, we seek for T-periodic solutions where T=$2\pi/\omega$, and analyze their orbital stability using Poincaré map and Floquet theory. Upon varying the actuation frequency ω and the stiffness ratio k_1/k_2, we find pitchfork bifurcations where the symmetric periodic solution oscillating about mean values $\bar{\phi}_i = 0$ becomes unstable (dashed lines) and a pair of stable asymmetric solution branches evolve. The results demonstrate rich dynamic behavior of periodic solution multiplicity, which also enables steering the swimmer’s path curvature by controlling its input frequency.



(a) Purcell’s three link microswimmer model with actuated-elastic joints. Bifurcation diagrams of periodic solutions branches, the mean value of joint angle $\bar{\phi}_1$ as a function of varying parameter. Solid curves denote stable periodic solutions and dashed lines denote unstable ones. **(b)** Case of 0<k_1≫k_2, ε_2=0, varying frequency ω. **(c)** Case of fixed frequency ω and stiffness k_2, and ε_2=0, varying stiffness k_1.

References

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[3] Passov (Gutman), E., & Or, Y. (2012). Dynamics of Purcell’s three-link microswimmer with a passive elastic tail. *The European Physical Journal E, 35*(8), 1-9.