Nonlinear dynamics and bifurcations of a planar undulating magnetic microswimmer

Jithu Paul[*], Yizhar Or[*] and Oleg V. Gendelman[*]

[*] *Faculty of Mechanical Engineering, Technion – Israel Institute of Technology, Technion City, Haifa 3200003, Israel*

Abstract. *A magnetically-actuated microswimmer model, motivated from biological microorganisms, which has two links representing a tail and a magnetized head is studied numerically and analytically. The links are connected by a passive elastic joint and the microswimmer is actuated by an external time –periodic magnetic field oscillating in plane. This simple system is very rich in dynamics and we identified that it may have co-existing periodic solutions, symmetric as well as asymmetric, and stability transitions with subcritical pitchfork bifurcations, induced by the system's parametric excitation.*

Introduction

A leading concept for nano-swimmers actuation is using planar time-varying external magnetic field [1] which can be set to be $\mathbf{B}(t)=\gamma\hat{\mathbf{x}}+\beta\sin(\Omega\tau)\hat{\mathbf{y}}$ where $\beta,\gamma\geq 0$ are constants. A simple theoretical model for studying the planar locomotion of such swimmer is the two-link model proposed in [2], see Fig. 1. This model consists of two rigid links connected by a passive elastic joint represented as a torsion spring, and one of the links (the "head") is magnetized along its longitudinal axis. The analysis in [2] focused on the case of small oscillations $\beta\ll\gamma$ and conducted asymptotic analysis of the motion in which the swimmer oscillates about and swims along $+\hat{\mathbf{x}}$ direction, which is a stable periodic solution. The analysis showed that there exist optimal actuation frequencies Ω for maximizing the mean speed or displacement per cycle. In this work, we revisit the two-link model in [2] and extend the analysis to cases of large oscillations $\beta>\gamma$ and even $\gamma=0$, and study also the "backward" solution where the swimmer oscillates about and swims along $-\hat{\mathbf{x}}$ direction. While this swimmer's orientation is statically unstable (for $\beta=0,\gamma>0$), we find that for $\beta\neq 0$, this gives a periodic solution, which undergoes stability transition and subcritical pitchfork bifurcation upon varying amplitude β and frequency Ω of the magnetic field's input. We analyze the backward solution numerically as well as analytically using asymptotic expansion and harmonic balance. Under small-angle expansion, the system's dynamics can be reduced to a nonlinear 2nd order differential equation with parametric excitation, which resembles the well-known Kapitza pendulum system [3]. Finally, we show optimization of the swimmer's net motion with respect to both β and Ω.

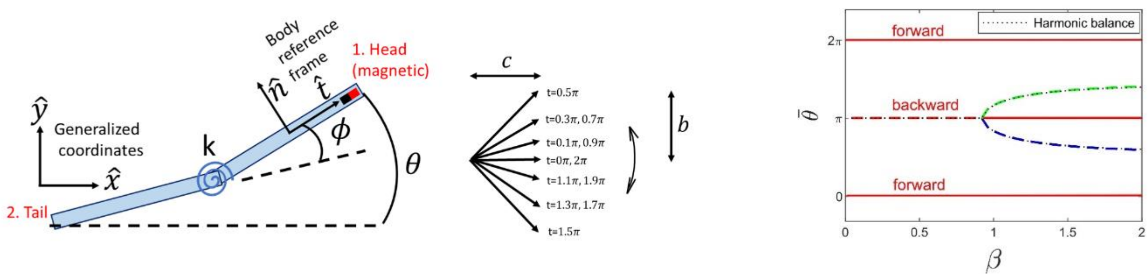

Figure 1: The model system and the bifurcation plot which comparing numerical (color lines) and analytical findings.

Results and Discussion

The dynamics of the microswimmer propulsion in the backward direction gives very interesting findings. In the forward direction, the motion is always stable, whereas in the backward direction ($\bar{\theta}=\pi$) the swimmer shows stability transition with subcritical pitchfork bifurcation, upon varying a single parameter out of β,γ,ω. The swimmer can go faster in the backward direction than the forward direction and nonzero net propulsion exists for the case $\gamma=0$. The parameter γ can be tuned to obtain the optimum velocity or displacement in the stable region, which calls for the scope of an experimental validation and gives hint towards its engineering applications in the future. Again, γ is a very sensitive parameter in the system and the dynamics at $\gamma\to 0$ needs further investigation to get a full picture of the nonlinear dynamics in the domain. The numerical approach successfully calculated the stability, bifurcation and optimum values of the swimmer's motion for the fixed point around $[\pi;0]$, for different range of parameters. The harmonic balance approach, very well predicts the symmetric and asymmetric branches of the bifurcation, and the condition for stability transition.

References

[1] R. Dreyfus, J. Baudry, M. L. Roper, M. Fermigier, H. A. Stone, and J. Bibette (2005) *Nature* **437**, 7060.

[2] E. Gutman and Y. Or (2014) *Physical Review E* **90**, 01301.

[3] Y. Harduf, D. Jin, Y. Or, and L. Zhang (2018) *Soft Robot*. **5**, 389; P. L. Kapitza.